Ξεναρμονία Η΄

Xenharmonic Bulletin № 8

JULY, 1976

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Back Issues:

In a subject such as xenharmonics, where no recognized communication channels have existed, it should be quite understandable that many people will be "coming in late."

At the time of writing, some back issues of XENHARMONIC BULLETIN are still available, particularly ## 5 & 6, at $1 each.

Nos. 1 and 2 will be reprinted in a new, combined format--details on request.

Other issues will be supplied on a copied-to-order basis, and prices for them will depend upon what is being currently charged for photocopying.

Many of you will want broader coverage of the field, i.e. to know what others in non-twelve-dom are doing, so might be interested in writing to XENHARMONIKÔN, the journal into which most issues of this bulletin will be bound. The address is:

Dr. John H. Chalmers, Jr.
Department of Biochemistry
Baylor College of Medicine
Texas Medical Center
Houston TX 77030.

Dr. Chalmers is planning a more extensive publication program for the future, which will involve monographs and possibly other relevant educational materials.

To order separate copies of this XENHARMONIC BULLETIN, write to Ivor Darreg at 349½ W. California Avenue, Glendale, CA. 91203.

Besides the Xenharmonic Bulletin, Ivor Darreg has issued a number of articles, leaflets, and pamphlets, as well as composition lists and music scores, conventional as well as xenharmonic. Sheet music is supplied on a "to-order" basis, by diazo prints.

CONTENTS:

This is as good a time and place as any, to give a résumé of the contents of the seven previous XENHARMONIC BULLETINS:

Back Issues of Xenharmonic Bulletin:

No. 1. March 1966
Change in attitude at that time, hence reasons for starting the Bulletin. Electronic instruments and computer music—new possibilities thereby.

No. 2. May 1974
Further progress summarized.
Second-order or tempered temperaments: widespread use of systems which deviate from theoretical equal tempering.
Tables: Intervals of importance in theory; the Harmonic Series.

No. 3. November 1974
General Remarks & Updating.
Review of Harry Partch's GENESIS OF A MUSIC, 2nd Edition, enlarged.
Appraisal of the 24-tone equal temperament, or Quartertone System.
What the Piano did for and against quartertones; Quartertone Notations; Good and Bad Instruments for demonstrating or Composing in the Quartertone System.
Guitar Re-Fretting Table for the 19-tone System in the most popular size, 650 mm active string-length.

No. 4. February 1975
Darreg's wall-charts of comparisons of Temperaments and Just Intonation, described.
Harmonics, Overtones, and Such.
Metric measurements for musical instruments.
"Illegal" Sharps and Flats: Microtonal Deviations and Tone-Bending in actual Performance but not written down.
Review: Music Teaching Text: George Whitman's INTRODUCTION TO MICROTONAL MUSIC (along lines of Hába System).
Review: UCLA Concert of Harry Partch's THE BEWITCHED.
22-tone Organ piece: ON THE ENHARMONIC TETRACHORD. 19-tone Guitar Piece: PRELUDE IN E MINOR.

(continued on Next Page)

Back Issues of the Xenharmonic Bulletin
Résumé, Continued:

No. 5. May 1975
New Moods. Article on the emotional effects and uses of different non-12 systems, an important reason for having several non-12 tunings.
Descriptions of Darreg's new instruments. Steel-guitar type, in just intonation throughout.

No. 6. September 1975
The Notation Question: Some of the many many ways of writing down Non--Twelvular Compositions.
17-tone composition: Pentatonic Study. 17- or 34-tone composition: LULLABY FOR A BABY COMPUTER.

No. 7. April 1976
Invitation for readers to comment.
FRETTING AND FRET-CHARTS: How to fret or re-fret guitars etc. to new scales; how to make charts for steel guitars and similar fretless instruments; FRETTING TABLES for the popular 650 mm active string-length, approximately 25-5/8", for the following systems: 17, 19, 22, 24, 31, and 34. Of course, 24 includes 12.
Description of the Hobnailed Newel Post, a new justly-intoned instrument with 64 strings, 16 on each side. Because of its length, 16 pages, the music score for four guitar parts in the twenty-two-tone system, COLORLESS GREEN IDEAS SLEEP FURIOUSLY, will be sold separately and done only to order.

Xenharmonic Bulletins 1 and 2 were bound into Xenharmonikôn 1;
X. Btn. 3 into Xenharmonikôn 2;
X. Btn. 4 into Xenharmonikôn 3;
X. Btns. 5 and 6 into Xenharmonikôn 4;
X. Btn. 7 into Xenharmonikôn 5.

During the last 30 years, Ivor Darreg has produced a long list of publications, a few of which bear upon xenharmonics and musical theory. The revision and updating of some of this is planned as time and circumstances will allow.

In addition, there are xenharmonic scores, but most of the music will be left in audio form as tape recordings, which can be duplicated and copied, again, as time and circumstances permit.

479 001600

Phone number? No, not quite right for that. Government debt? I don't blame you for asking, but not that either. Well what, then? Factorial 12.

479 001 600 is the number of different orders in which 12 things can be arranged, provided of course that there are no duplications. In 19th-century England, this number was the theoretical number of changes that could be rung on twelve bells, although the rules of change-ringing might cut that down somewhat. It would take almost 38 years to ring that many changes, no time out for coffee-breaks, eating, or sleeping.

479 001 600 is also the number of Schönbergian "tone-rows" possible, and so the 12-tone serialists bring up this number once in a while to boast how enormous the resources of 12-tone serialism are.

So long as it remains on the "aboutness" level, and so long as attention is riveted on the black notes on white paper, this seems reasonable.

However, these millions of tone-rows do not sound as different as they **look.** *There are several reasons for this, one being that so many of the resources of ordinary music had to be renounced.*

On top of that, anything that will imply traditional melody and harmony, to which the public has been conditioned, has to be avoided. The listener, more than the performer or composer, has to overcome his years of conditioning, and this creates a subconscious strain.

As pointed out in Xenharmonic Bulletin № 5, the mood of the 12-tone temperament is restless. Atonality and serialism increase this restlessness by avoiding the tonic and certain other concepts.

This restricts their powers of expression, and makes the abstract mathematical argument about factorial-12 much less cogent.

479 001 600 continued

This should serve as a warning to us that mere abstract calculations or tabulations of the possibilities of this or that tuning-system may be misleading.

Because 24-tone has more sounds per octave does NOT necessarily mean that it has more musical resources than 19 or 22.

How these systems and methods actually SOUND is what counts, and this cannot be calculated in advance, any more than you could evaluate paintings without seeing them, by means of calculations based on the cost of paint, canvas, brushes, etc.

When someone writesan article or book either for or against a tuning-system, they consult everybody except those composers who have tried it out!

Over and over again, with the most tiresome regularity, the scholarly articles and books bring up the difficulty of writing this or that non-12 system, or how troublesome the extra tones would be for the KEYBOARD performer, or the contention, which has considerable truth to it, that Bach, Haydn, Mozart, et al. got along very well with what they had at their fingertips, which usually included keyboard instruments with only 12 tones per octave.

That two centuries have passed, and that the obvious resources for composers have been exploited, and that the danger of a new composition merely rehashing the clichés of yesteryear has continually increased, is carefully ignored.

The composer isn't supposed to have any pride anymore, but instead to be content to being reduced to the status of an arranger, or pasticcio expert, or a(subconscious) quotationist. If that is to be the future state of affairs, you might as well replace him with a computer right now, and get the miserable business over with.

If the rules deduced from 18th-or 19th-century compositional practice are to be slavishly followed forevermore, just for the sake of performers and instrument-manufacturers, then the computer can obey them far more perfectly than any mere human being could ever hope to: this has been proved already.

The rules of 12-tone serialism are to some extent a complementary reflection of the earlier rules just mentioned. In fact, they depend on them, in a strange sort of way, and they are all the more strict, since they are arbitrarily con-structed by people conscious of logical principles and patterning, rather than being passively deduced from a certain period's composers' actual practice.

Happily, there is a "free atonality" which doesn't accept all the new strict rules, and isn't tied down to the set of 479 001 600 tone-rows either. We can at least tell these persons about xenharmonics and the enormous variety of systems suitable for atonality, but so far virgin territory.

Whether it was practical or not in 1705 to build a harpsichord with more than 12 tones per octave and keep it well tuned, has no bearing on what should be composed today.

Whether a piano tuner can or cannot, will or will not, tune a keyboard instrument to the 13, 19, 18, 21, 23, 22, or 31 system, no longer matters in this Electronic Age, even though it mattered as recently as 1945 or 1963.

Whether today's performers can or cannot, will or will not, perform non-12-tone music must not stop a composer from using non-12 systems: it is no longer necessary to interfere with the Musical Establishment's prejudices in order to hear a new composition. They can be left alone to continue their old conditioned, programmed ways.

That is to say, the XENHARMONIC BULLETIN is not a crusade to abolish the 12-tone system. How could it be, when I have been involved in it for 53 years and have a long list of piano and organ compositions? I do not have to burn scores or wreck instruments as some people actually did, because

479 001 600 concluded

I am adding to the status quo, not uprooting everybody for the sake of "revolution".

With xenharmonics, there is something with which to CONTRAST 12-tone. So long as you remain inside 12-tone tuning, you cannot be aware of its personality or mood. Discovering this contrast is in itself a novel, unsuspected experience.

Nobody could predict it by silent, visual mathematical calculations.

Serialism is valuable for the peculiar way it sounds, and for the range of emotional and other effects it can produce. It is not limited to the 12-tone temperament, and it should be explored in non-dodecaphonic systems. That is to say, don't take the preceding paragraphs as a putdown of atonal serialism, but as an invitation to join the xenharmonic movement and hear some non-clichés.

If you happen to be an atonal or serial composer, why not try "mapping" something you already have written, onto a scale other than 12-tone equal? It might be a pleasant surprise.

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I've Built It or Bought It; Now How Do I Tune It?

Some people are going to feel that I am presuming too much here: if so many otherwise talented and capable persons can't master the 12-tone tuning routine for orthodox pianos and organs, how dare I expect anybody to master a routine for microtones?

Frankly I don't. When it takes 1½ to 2 hours for an ordinary tuner to tune a piano (more if the customer is watching television), and pipe-organ tuning can run into a matter of days, you cannot be blamed for getting frightened at the difficulty of tuning some instrument to the 41- or 53-tone system.

XENHARMONIC BULLETIN NO.7 has already given one way of escaping this problem: Use fretted instruments, and the mathematical calculations which place the frets ensure correct tuning once and for all at the time the instrument is built or modified.

Well enough as far as it goes; but fretting an instrument to more than 31 to the octave runs into all kinds of difficulties, and *playing the instrument after you get it made* becomes increasingly difficult; and this problem escalates rapidly!

The previous article dealt with fretting to the various *equal temperaments*, not to Pythagorean or just intonation or something in the meantone family. A justly-intoned fretted instrument will require at least some frets spaced a comma (81:80) apart, and possibly other small intervals—the fret-spacing for a comma is equivalent to that for a 55-tone equal temperament, so if you happen to be a guitarist, I am fairly confident this will make you shudder.

Inevitable errors in placing the frets, and other errors entailed by pressing strings down against such frets, also limit the practicable fineness of dividing the octave by this means. So, while the fretting or re-fretting approach is ideal for the beginner in xenharmonics, and for practical performance in a number of systems affording varied new moods, it will not take care of everything.

Systems with smaller numbers of tones which do not have good approximations to the conventional harmonic intervals (13-tone, for instance) will be well served by fretting, whether on fretted instruments themselves, or using such instruments as secondary standards to tune others to.

At first blush, it might appear that the new *electronic tuning devices* had solved all these problems. I wish they did! I wouldn't have to write this article. For one thing, most of them are quite expensive. Prices may come down eventually, but we're talking about *now*.

In many cases, such devices will have to be rented or borrowed, or the instrument taken to where they are. What do we do in between times, or when a xenharmonic instrument is used somewhere that power lines are not available?

Some of these devices are biassed in favor of the twelve-tone equal temperament; others may be more or less tied to a certain starting-pitch, usually A at 440 Hz.

Two complementary electronic devices are becoming more generally available and less expensive in recent years: the *digital readout frequency-counter* and the *frequency synthesizer*.

The frequency-counter was developed to measure radio frequencies, at first in the broadcast range of around a million counts per second, and then extended into the billions, as manufacturing techniques got better and better. One part per million accuracy is commonplace.

How Do I Tune ?

Frequency counters work well with sustained musical tones, such as those from an organ, but would be worthless to determine the pitch of a high note from a xylophone or banjo.

A frequency synthesizer can be set by dials, pushbuttons, adding-machine-type keys, or similar means to a desired frrquency, and then it will output this frequency as a low-level signal, which can then be converted to visible or audible form for comparison with whatever is to be tuned or checked.

Its accuracy is to some degree spoiled by the conditions of the comparison: tuning by ear or with some visual aid. However, this error of comparison may not be too serious in the practical musical situation.

Since musicians do not think in terms of '220 Hz' but rather of 'A below Middle C' or 'second ledger line below the treble staff', *frequency tables* are necessary in order to use either frequency counters or frequency synthesizers.
This is true also of some of the other tuning devices and electronic aids for tuners.

Some recent electronic organs have a fixed 12 tone tuning device built right into their tone-generator circuitry, so they never have to be tuned; everything is 'locked' in. Retuning to non-12 systems is thus out of the question. So be forewarned if you plan on getting organs for xenharmonic purposes!
This is probably the best place for me to repeat my pet protest: *One* tuning-system is not enough, now that 12 has been worn threadbare. If you did not agree to this, you probably wouldn't be reading the present article.

Very well: *Two* tuning systems, i.e. 12 and 31, or 12 and 19, are not enough *either*. In order to encompass the full range of moods, a number of different systems must be available. You don't need every system; there are too many; but my composing so far and my trials with over 200 listeners indicate that at least ten systems will prove sufficiently different in effect to make it worthwhile to have such a variety.

I don't expect you to have time for practicing and performing in many systems, nor the funds for a whole arsenal of 'dedicated' (one-system each) instruments, but I *can* ask you to collect recordings in as many different systems as possible.

Each system has a characteristic *mood*, and the choice of moods will have some connection with your personality and that of the composer, along with what has to be expressed in music at the particular point in time.

I don't think I am being unreasonable if I ask that at least four (*which* four is strictly up to you) systems be available in the well-appointed xenharmonic studio.

Please note that two systems like 17 and 34, where 34 *contains* 17, or 12 and 36, where 36 *contains* 3 circles of 12 fifths each, ought to count as 1½ systems rather than 2. After all, few people would argue that the whole-tone scale, the 6-tone system contained within the ordinary 12-tone one, would count as a completely independent second tuning-system.

The preprogramming of tuning-systems is already reality: the Scalatron, for instance, can store the necessary information required to retune it to any of several systems at the touch of a button. No doubt this principle will be extensively applied in the future.

Again, I have to say that we are talking about *Now*. How does one tune an instrument to the new scales in the home, or a small studio? What is feasible or practical when you are just beginning to explore this *terra incognita*?

We read constantly of feats of fantastic, incredible precision. The man-in-the-street is appalled by abstruse scientific statements about extremely accurate measurements, and often wants to discount them as though they were exaggerated advertising claims.

You really can't blame him, there has been so much misuse of extra decimal places where they do not belong. So, with healthy skepticism, let us ask: How much precision does xenharmonics really call for?

I have to weaselize here: it all depends.
What system do you have in mind? What instrument? Above all, how fast is the music being played? What tone-quality? Is the scale, and/or the music, harmonic or inharmonic? Where does the main interest lie, in the pitch-range?: middle register, high treble like the glockenspiel, low-bass like organ pedalboard, or tenor/alto like the viola?

The way it works out is that a 31-tone organ is very much worth while, but a 31-tone banjo would be a ridiculous waste of time; a 17-tone banjo would be dynamic and brilliant, but a 17-tone harmonium or organ would be unbearably harsh.

We have had several generations of people influenced by the piano, and an enormous piano literature holding things in place till quite recently. This means *inertia* (or momentum, if you happen to be in favor of the status-quo situation) and *subconscious biasses*.

How Do I Tune?

On the one hand, the convenience of tuning or having tuned, two pianos a quartertone apart, encourages many people to experiment with the 24-tone equal temperament. On the other hand, the piano's timbre is such that all the new intervals of the quartertone system sound 'funny' or queer or odd, and thus most of the above-mentioned experimenters abandon their quest.

If the guitar had been as popular during the periods of quartertone-piano experimentation (such as the mid-1920's) as it is today, and if electric guitars as loud as pianos had been available then, the quartertone story would have come out quite differently!

For the purposes of this article, that means that I can be selective, and not spend too much time on unsuitable combinations such as a 22-tone piano and how to tune same, or how to fret your mandolin, ukulele, or banjo to 31-tone.

I don't have to take up the subject of how pipe-organs are tuned, because few people could afford a xenharmonic pipe-organ nowadays.

I definitely do not want to encourage anybody to tune an *incomplete* set of a system's tones, such as a piano to only twelve pitches of the meantone temperament, 12 out of 31, 12 out of 19, nor even 19 out of 31.

There has been far too much of that sort of thing. What it amounts to, is that those who do not compose any new pieces are forbidding composers to explore such tuning-systems. Usually the motives for say, 12 tones only of the meantone system, were legitimate enough at the time and within the context in which they were acted upon, but this does not help the cause of xenharmonic progress. The implication of a 12--out-of-meantone tuning is that the 'wolf' is *an inevitable part* of the meantone system, when it is nothing of the kind. Instead, it is a compulsory wrong note caused by the potent combination of mechanical difficulties with expense.

The wolf (wrong interval resulting from the circle of *12 meantone fifths* failing to close) has even been extolled and praised by antiquarians wanting early keyboard music to be authentically performed. On the surface, this seems innocent enough, but as a composer, I can't imagine anybody wanting a diminished sixth such as G-sharp / E-flat or an augmented third such as A--flat / C-sharp while *writing* the normal fifths and fourths such as A-flat/E-flat or G-sharp/C-sharp in the score.

When instrument-builders, dealers, performers, historians, and theorists gang up on today's composers and tell them they must not write or play or hear G-sharp and A-flat sounded simultaneously on a meantone, 31-, or 19-tone instrument, that is going too far! Conversely, there isn't much use in a modern composer trying out a retuned piano on which there still are only 12 different pitch-classes.

You are just wasting your time and moreover frustrating yourself if you try to explore a more-than-twelve-tone system on an instrument whach only has 12 tones. You can't imagine the difference between G-sharp and A-flat in certain systems unless you can *hear* it and *play* it, and then take advantage of it by actually using it in new compositions. I don't care three broken jellybeans whether certain 17th- or 18th-century musicians had their keyboard instruments tuned in this or that version of meantone or other temperament, because this did not necessarily determine how violins or viols were played, and no matter what they did then, it must not influence how 1976 composers write new music on new instruments.

Getting back to the topic of precision, Bach and his contemporaries didn't have accurate frequency-standards nor accurate frequency-measuring equipment, so even though certain temperaments could be accurately *calculated* by the theorists, that does not mean that clavichords, harpsichords, etc. would present these tunings in the stable precision form that today's electronic organs and computer performances can.

I can't help thinking of the Hollywood movie studio that had to scrap an expensive sequence because a couple of the Roman soldiers in the battle-scene had their wristwatches on. This is the kind of anachronism some music theorists indulge in when they extrapolate 1976 precision back to Renaissance or even Ancient Greek instruments.

Another similar matter concerns the materials and tools available before the 19th-century industrial revolution and the 20th-century perfecting of mass production and interchangeable parts and refinement of measuring-standards.

Today's steel music wire, used for all kinds of springs and parts of non-musical mechanisms, and very similar to drill-rod in character, is quite a different sort of thing from 17th- or 18th century harpsichord wire, and extremely unlike the handmade gut or other strings used by the Ancient Greeks!

Today's nylon guitar and harp strings are fresh from the modern chemical laboratories.

Who knows what materials the researchers will perfect next year or next decade?

So for materials going into other kinds of musical instruments. For instance, the pipe--organs of Bach's time were not blown with turbofans driven by electric motors.

The temptation to extrapolate our precision back to earlier instruments is hard to resist.

Because an electronic organ or a computer performance can now have precision beyond any human being's ability to hear, there is even greater

How Do I Tune?

danger that one will demand impossible accuracies from other kinds of instruments that will be used to play the new scales.

It's your valuable and irreplaceable time, money, and energy that I'm talking about here. I am assuming that readers of this article are considering and mulling over which scale-systems and which instruments they are going to make or use, and naturally one is afraid to have expensive 'white elephants' around.

Some of you are already familiar with the standard routines for setting the 12-tone equal temperament on pianos and organs. Others of you will have read about it, or watched and listened to a tuner at work.

Nearly all public libraries have books on piano-tuning and sometimes also on organ-tuning. Some of this information is relevant to us who are going in for xenharmonics; other of it need not bother us—such as the peculiarities of wound and double-wound bass strings on pianos, or the special skills involved in tuning the several kinds of organ pipes and types of reeds.

You may be one of those who say "I don't need to learn how to tune by ear; I bought an Elektronik Ghizzmo Super DeLux." Mark VI, no less. Or, "I'm a guitarist. Why not let the frets do it for me?" Or, "Let the computer do it!"

It is possible to get along without knowing any tuning-routines at all, but there are some real advantages to studying them. You may want to check the tuning of an instrument: Has it drifted? Are the octaves stretched or shrunk? Was it put in an equal or unequal temperament? If just, is it just throughout?

Even though you have a tuning-device, there may be some unexpected chance to check an instrument—and no!—you don't have it along.

Then there's the financial angle: even in these days of unemployment and all, tuning is in demand, and knowing the 12-tone routine can be well worth the trouble.

No point trying to take the place of the standard books on the subject here; but some discussion of the orthodox twelve-tone routines is quite relevant.

Beginners in tuning often ask: "Why not just get a set of tuning forks for all the 12 semitones, so that there will be no need to bother with a tuning-routine at all?" Such forks *are* available; so are sets of reeds and other fixed tuned devices.

If you depend on gadgetry to this extent, the gadget will own you, and you will be its slave. This can be quite embarrassing on occasion, for a mere machine, or the lack of same, to humiliate you in public. So it's a matter of pride.

An obvious disadvantage of tuning by ear with a routine is that it is imprecise; every step is prone to error, and it takes several months to practice and learn a routine.

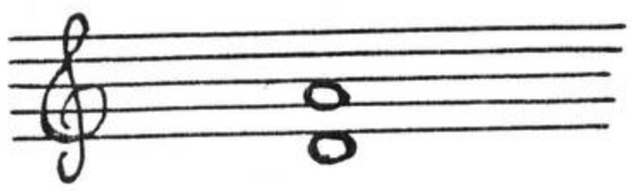
The compelling advantage is that once memorized, it becomes second nature, and it is valid for a number of different instruments and regardless of the actual pitch, whether above or below standard. The routines are to some degree *flexible;* slight variations are sometimes made.

Pianos often are too far below pitch to be safely raised in one operation. Orchestras often demand a higher pitch than normal, such as A at 442 or 444 Hz. Many organs are built for A = 435 Hz and cannot be raised, or there are chimes or 'harp' stops that cannot be altered. Musical historians rightfully demand that certain compositions be performed at original pitch.

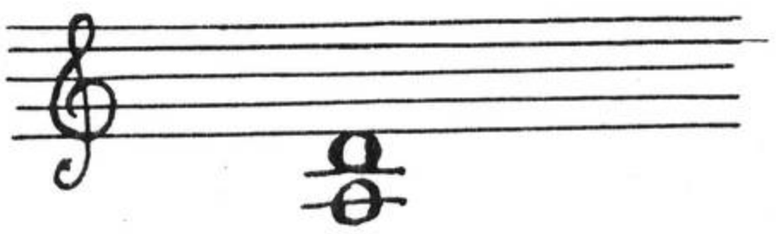
The 12-tone routines are based upon *beat tables* showing the number of beats per second for certain intervals within the range used for setting the temperament. For pianos this is most often F below Middle C to F above Middle C; for organs this is usually Middle C to the C third space treble staff; a useful compromise is the A of the tuning-fork down to the A an octave below, or 440 to 220 Hz at usual standard pitch.

Beat-tables for several systems will be found on Pages 17 ff.

Let us consider the beat-rate of the fifth D to A or 293.66 Hz and 440.00 Hz.



The same beat-rate belongs to the fourth below it, A – D or 220 and 293.66.



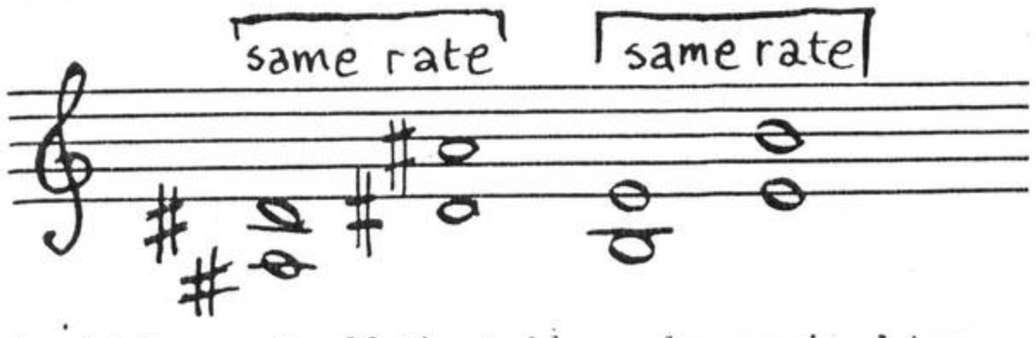
The table says 0.99 beat per second, which of course means just about once a second outside the laboratory.

This beat-rate is for the 12-tone-tempered intervals shown above in conventional notation, and be it noted that a common beginners' mistake is to get the right beat-rate but the wrong side of just, i.e. the fifth sharp instead of flat. It's only 2 cents, 1/600 of an octave, so this is understandable!

HOW DO I TUNE?

With the aid of watches, pendulums, and now various electronic apparatus, as well as metronomes, the would-be tuner can become familiar with one beat per second.

Theoretically, the beat-rate for other fourths and fifths, such as D-sharp – A-sharp or B – F,



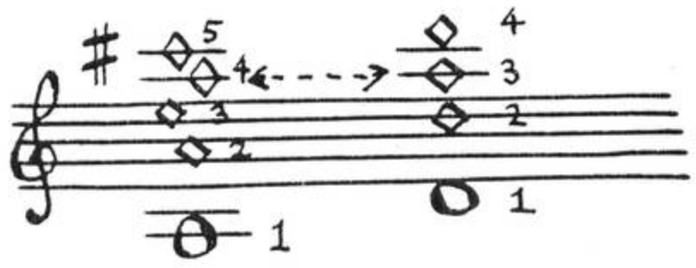
should be read off the table and memorized by practicing hearing it in time to some time-keeping device, but obviously this is too much for human memory, and such precision is useless in the real world out there anyhow.

So what actually happens? Why, one accepts a compromise: one uses a beat-rate in the middle of the tuning-octave for all intervals of that same class above or below it.

This obviously does not produce a perfect twelve-tone *equal* temperament. I call it a second-order temperament or a tempered temperament. But once you compromise by tempering at all, you've already 'sold out', so why quibble? No one will ever know.

Thus the errors in guitar-fretting spoken of in previous articles have their counterpart in the errors inherent in the customary organ- and piano tuning-routines. Furthermore, these have still another counterpart in the methods used to approximate tempered tuning in electronic instruments, or in digital-computer synthesis of music.

How was the table calculated? By finding the lowest pair of coincident harmonics the interval would have if it were just, and taking the amount by which the harmonics of the tempered tones fail to coincide—their difference.



In the above example, the Fourth A–D, beloved of piano-tuners, if it is just, the 4th harmonic of the A coincides at 880 Hz with the 3rd harmonic of the D, and there are no beats. In the twelve-tone case, the D is about 1/3 Hz sharper than just, and so its 3rd harmonic is three times its frequency and therefore about 1 Hz sharper than the 880 Hz 4th harmonic of the A at 220 Hz. The 880 and 881 are what beat.

How were the tempered pitches, upon which the beat-table was based, calculated? Through the use of logarithms, which are a kind of tempered arithmetic. So you mathematicians out there, don't you dare look down your noses at the mere musician. The tempering principle is everywhere.

The engineers' approximations and the tuners' routines are cousins-not-so-far-removed.

Put a slide-rule near a fretted fingerboard and see what I mean.

While a watchmaker or a quality-control inspector in an IC factory might find the standard tuning-routines too crude for his taste, the people at the average repair shop or contracting firm would think of them as rather fussy.

If you decide to learn tuning-routines, don't try for more precision than they can deliver.

While Alexander J. Ellis did give a routine for the 53-tone temperament in his famous Appendix to Helmholtz's *Sensations of Tone*, that does *not* mean that anybody should try to learn and use such a routine in this day and age. Back then, it was the only game in town; today, there are so many alternatives that no-one who could afford a 53-tone instrument would put up with the tedium, inaccuracy, and boredom of such a procedure. Its only value now would be to check a few notes suspected of having drifted.

The user of some kind of tuning-device might want to check the accuracy of transfer of each tuning-device note to the instrument, or for the sake of greater objectivity, have an assistant do so. In this case, a beat-table would help, so beat-tables for many-toned systems have their uses.

Beat-tables will be published in this and future issues of *Xenharmonic Bulletin*, but please don't expect everything at once!

Fortunately, it is not necessary and not even feasible to construct tuning-routines for all the usable tuning-systems—we would never get done if it were practicable, and nobody would master very many routines anyhow.

We might point out that the beat table for meantone temperament will differ slightly from that for the 31-tone equal temperament, and in theory a meantone beat-table would never be completed, since it contains an infinite number of notes and intervals.

Inevitable errors in tuning and the very nature of tuning-routines would, however, make any of the existing meantone routines do just as well for 31-tone—to close the circle of 31 fifths correctly would require some kind of help besides the routine itself. Meantone fifths and fourths beat fast enough that one beat-rate will not do for the entire octave as it will for the twelve-tone routines.

N.B.: This remark applies also to the 19 and 22 systems!

How do I Tune?

There are very many ways in which to vary the process of tuning 31-tone or meantone, and still have a tolerable result. Much ingenuity has been squandered on trying to cover up or mollify the 'wolf' produced by the 12-out-of-meantone scheme, and there are a host of unequal temperaments, with wolves in or out of sheep's clothing.

Surely you would save time and headache and annoyance by taking the pitches from a 31-tone electric guitar and then adjusting any mistakes in transferring the tuning by means of a beat-table. The only way to get greater accuracy is the electronic tuning-device.

That is to say, balance the value of your own time learning and practicing the 31-tone tuning routine against the cost of having a guitar refretted to the 31-tone scale with my table, and against the cost of an electronic tuning device.

This may help you to understand why the 31-tone innovators have had such a hard time of it for so long, despite Huygens' stature, and yet the meantone traditionalists are doing well enough since the harpsichord revival.

12 tones of meantone represent a slightly easier tuning routine than 12-tone equal, whereas the remoter reaches of the 31-tone cycle, when attained by ear in the old-fashioned way, have almost certainly picked up and accumulated excessive errors.

It is true that some of these remote tones can be checked by chains of major thirds and/or sub-minor sevenths, but these also are harder to tune by ear. I must therefore put 31-tone into the class of systems very difficult to tune by ear-routine methods. This will also explain why piano manufacturers will hate you to pieces if you start talking about the 31 system. No tuner would have had the patience even if such pianos had been made. It hurts me to think about such a tuner's problems.

31ists should be most grateful to the electronic organ which unblocks these difficulties.

All right; where will by-ear-routines work?

Most important point: Routines-by-ear are limited to *only certain qualities of tone!* Certain harmonics must be present, and that beyond a minimum percentage.

Inharmonic partials must be held to a very small percentage. The tone must be sufficiently *sustained.* The timbre requirements differ for the different systems. This is too often ignored!

Twelve-tone equal temperament owes its success to the late-19th-century ideal of piano tone, and to such instruments as nylon-string guitars and soft flute-tone organ stops — and since the 1930's a variety of electronic instruments with deliberately-dull tones. Marimbas, xylophones, vibraphones, etc. have had some part in making the 12-tone thirds and sixths endurable. The electric-clock-motor inspired tone-wheel, driven through special gearing, used in the Hammond organ till quite recently, had a group of special features designed to make 12-tone temperament bearable: the *imitation-harmonics were tempered, and therefore could not include the 7th;* the use of tempered imitation-harmonics almost eliminated beats, so these tones could not be evaluated by a piano-tuner or organ-tuner in the regular manner; they are abnormal.

Such instruments (there was a transistor organ on this additive-tempered-synthesis principle) do not have the normal mood of the 12-tone temperament, so this has been a factor in conditioning the public for the last 35 years or such a matter.

'Background' music systems in stores, offices, factories, etc. attain their blandness (cf. Muzak's advertising of 'heard but not listened to') by tone-controls reducing high-frequency components of the sounds and hence reducing beats and dissonances of the originally-normal tones employed.

This kind of dulled tone is not as serious a barrier to tuning-by-ear-routine, but it can make tuning of some electronic organs difficult. It could make piano-tuning difficult when the instrument has tired listless aged strings.

Xylophones could never be tuned by a routine because the harmonics to beat are almost absent, and even if they were present the tone does not sustain enough. If someone built a keyboard banjo, the tones would have the right harmonics, but routine-tuning would be out of the question, since no tone would last long enough to count the beats.

Metallophones and bell-like affairs such as the celesta or glockenspiel might sustain enough, but do not have the harmonics for using a tuning-routine—but there is a trick: if each of the two notes involved in one step of the routine could be *separately* electronically picked up and routed through *separate* distorting amplifiers on the 'fuzzbox' order, the necessary harmonics would be present.

Similar reasoning applies to the sinewave oscillators which experimenters sometimes use.

You could adapt a routine to *visual* tuning of such oscillators with an oscilloscope, but whether it would be worth the trouble is a moot point.

There is an important class of tones, such as clarinet and its imitations, square waves from tone generators, dividers, or synthesizers, and stopped organ-pipes, which lacks *even* harmonics.

How Do I Tune?

On most synthesizers these odd-harmonics-only tones are readily available, being derived from a square wave in most cases. Some modern electronic organs have them, often as the basic stuff from which all their other tones are made; while in other commercial electronic organs this 'hollow' or 'clarinet-like' or 'stopped flute-like' timbre is lacking. Generally, this timbre is considered to be an essential ingredient in organ ensemble. In the symphony orchestra and the wind band, it is closely approximated by the clarinets.

The pianist never has it, since so far as I know, pianos have never been made with the hammers striking the strings in the middle!

The electrical engineer is familiar with the odd-harmonics-only waveforms, because alternating-current machinery generates symmetrical waves, which could not be symmetrical if they contained even harmonics. Therefore hum picked up from power lines contains only odd harmonics until or unless it is distorted in some way, as by an amplifier.

At many music stores and toy-shops you can buy what is called a *song whistle,* which is essentially a stopped organ-pipe with a movable stopper controlled by some kind of handle. This is an inexpensive and pleasurable way to become familiar with odd-harmonics-only tones.

Such stops on a pipe organ cannot be used for setting the temperament; they have to be tuned from other stops by unisons. If you are a piano tuner and want to go into organs, pipe and/or electronic, remember this point!

Warning: There are electronic tuning-devices now on the market which output a square or other odd-harmonics-only wave. They are fine for tuning unisons by ear to them, but useless for tuning something else an octave above them, since there is no octave harmonic to beat with!

If you *must* use one of these handy doojiggers, you could pass the output through a distorter which would add the missing even harmonics.

The same remarks apply, even more so, to the electronic tuning-aids which output a *sine* wave, which is fundamental only with no upper partials at all. This needs a distorter to tune anything other than unisons by ear.

The reason I have to harp on this topic here is that the visually-oriented electronic expert has a love-affair with the sine wave, which can indeed be used to tune octaves or other simple musical intervals via Lissajous' figures on an oscilloscope and in certain other special visual apparatus. So he thinks it ought to work gorgeously jim-dandy for audible tuning too. Sorry about that, Mister, it never ain't can't don't!

Two sine-wave tones are endurable when they form the interval of a major seventh, whereas normal musical tones sounding this interval are dissonant. (Assuming a normal middle pitch-range, of course.)

To recap: Sine-wave tones, such as from tuning-forks or harmonic-free oscillators, can only be used to tune unisons by ear, although of course they would beat with the octave harmonic of a normal tone.

Odd-harmonics-only tones can be used to tune unisons or intervals whose ratios *do not contain an even number:* so they would beat for the major sixth, 3:5, but not for the major third 4:5 nor the minor third 5:6.

The above remarks, of course, apply to cases where both tones lack even harmonics. When one tone has them, the ratios having an even number can be tuned *if* the normal tone happens to be on the 'right side' but not otherwise.

What makes all the above so relevant, is that tuning-routines either set the temperament with fourths, fifths, and octaves—ratios 1:2, 2:3, and 3:4, all of which contain an even number, or they *check* with those intervals.

The ideal timbre for tuning by routine is not necessarily the ideal or even a desirable tone-quality for a particular musical purpose and that should be borne in mind if you are building an instrument to be tuned by ear.

For instance, the very characteristics which make it easier to set 12-tone temperament on a harmonium or reed-organ than on a piano, by the same token make 12-tone-tempered harmonium music less agreeable.

Those who would like to experiment with the special tone-qualities proposed for 19-tone instruments by Joseph Yasser in *Theory of Evolving Tonality*—timbres in which the 3rd and 6th harmonics would be totally absent—would not be able to use any practicable tuning-routine. No fifths, no fourths, no minor thirds, no major sixths. Minor sixths and major thirds would still be available to beat, but no way of checking them; so errors would pile up. One would either need an electronic tuning-device, or to transfer the tuning from another instrument of normal timbre.

Paradoxically, this article is appearing, and I have been constructing routines, just when they are declining in importance, due to the electronic tuning devices and the new ease of fretting, as well as new means of building electronic organs in such a way that they are their own programmable tuning devices.

HOW DO I TUNE?

Don't let this or anybody else's article discourage or prevent you from using those systems which cannot be tuned by routine, or for which routines are difficult. *Examples: 13, 14, or those with large numbers of tones.*

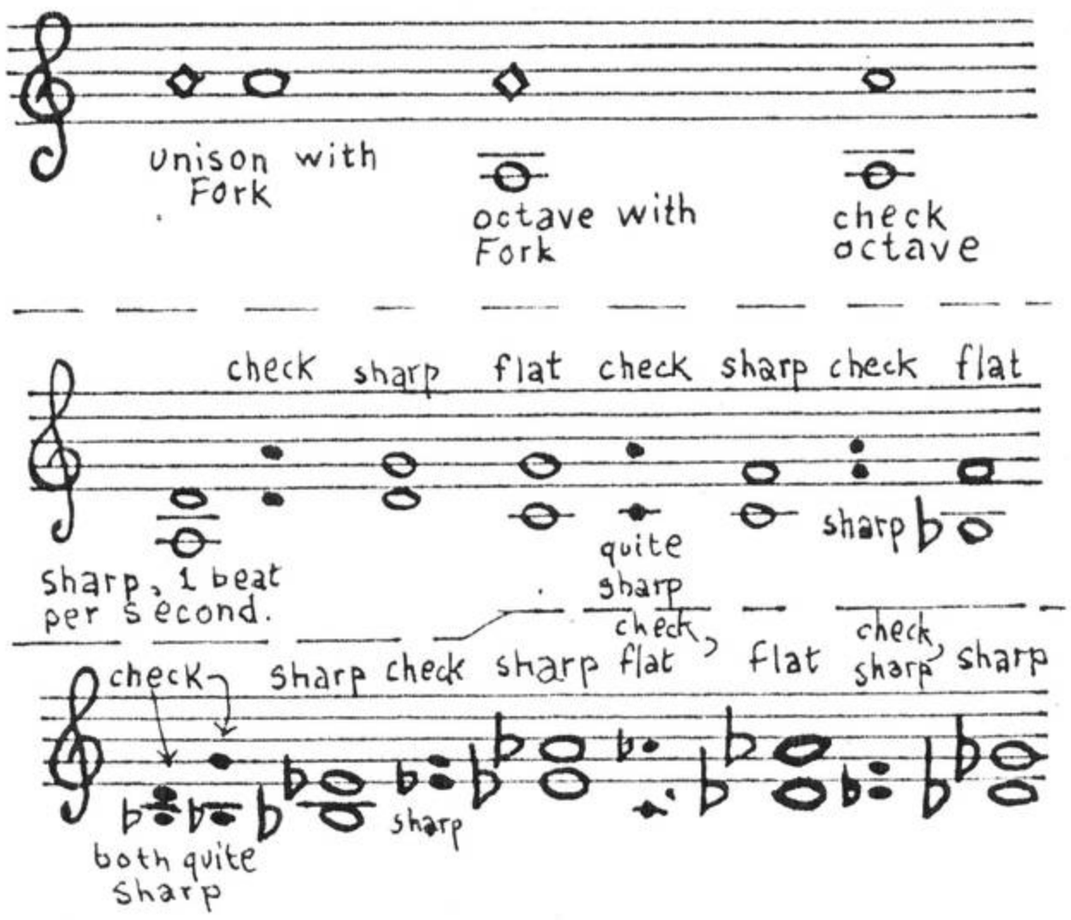
But the decline of routines will be very gradual, not sudden; and besides that, not only the routines themselves, but the way they are constructed and taught, are of considerable theoretical interest. They produce what I have dubbed *tempered temperaments.*

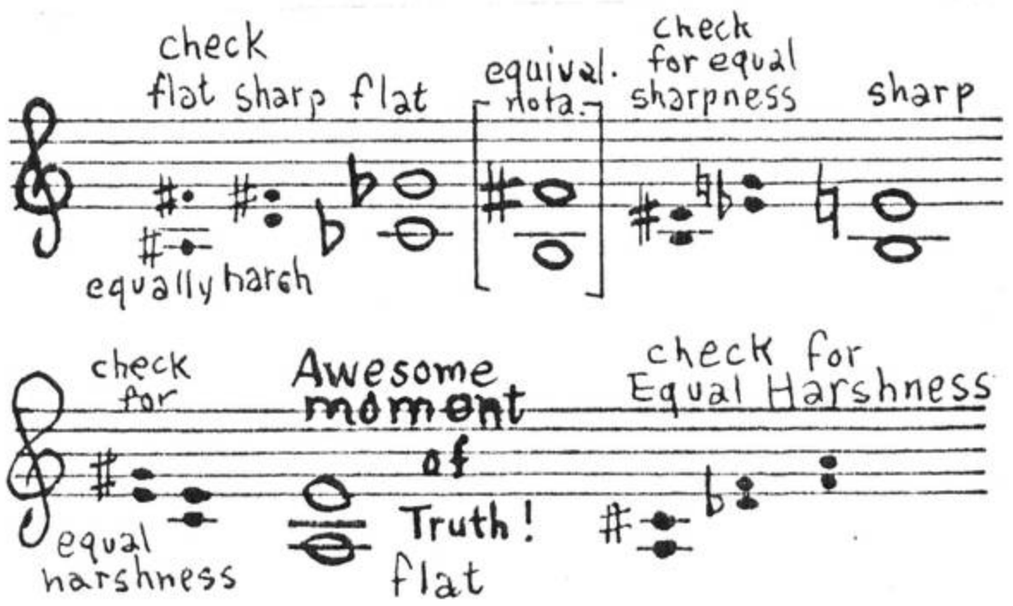
Since music is a sort of language of emotions, it is hardly strange to find all the practical, theoretical, and mathematical and engineering issues connected with musical instruments and the tuning thereof, beclouded by years of arguments over trivia, wild polemics, hurt and wounded pride, savage condemnations of this or that system, *petitio principii, argumenta ad hominem, trying to have it both ways at once,* and such-like. No, it would be strange if people were cold and unemotional about this subject.

It's now time to get down to brass tacks. The routine for 12-tone equal temperament has to come first, so that other routines can be compared with it. Fortunately, thank goodness, I don't have to go on at length: the libraries are full of books describing innumerable variations.

I give here the routine I follow in almost all cases. Before you snarl and snap at me, the choice of A 220 Hz to A 440 Hz as the tuning octave is a compromise between the F-to-F of piano specialists and the C-to-C of organ specialists, and is not being imposed or foisted on you if you prefer the other starting-points.

Use a tuning-fork or modern electronic substitute to sound a′=440 Hz (but this will work for other pitches, if such be necessary).





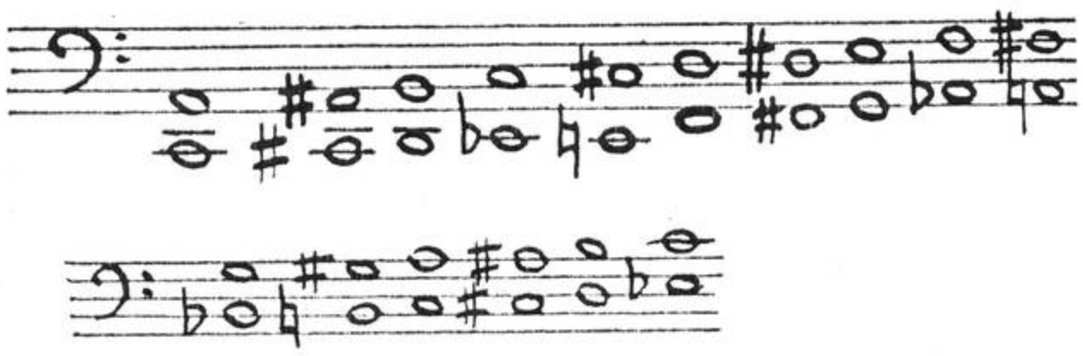
Ideally, the tuner would look up all the beat-rates in one of the published tables such as the 12-tone entries in the Quartertone Beat Table in this issue, and eventually memorize them. Actually, he memorizes some of them and the result is a tempered temperament which averages out.

The above order goes in a flatward direction around the circle of fifths, but there is no reason why you can't go in a sharpward direction if you prefer. It takes the average person a few months to close the circle on the first try, so don't get discouraged!

If you are tuning a master-oscillator-and--divider electronic organ, that's all there is to it; the octaves do themselves automatically. If you are doing a piano, harpsichord, or pipe-organ, you've just begun!

Tune the bass octaves first, then the treble end.

As soon as you have done the bass, check the major sixths chromatically from low C; for the beat-rates, which should rise gradually and quite evenly:



Don't get too mad if you goofed: the error may well be in the octaves—which is why somebody thought up this test in the first place. Even on the octave-divider organs, this is still the best range to check major sixths.

Be a nasty mean critic: have fun—or rather, *Schadenfreude.* Go around comparing the actual present tunings of 12-tone instruments with what the table says they should be. You won't have any friends left after this—certainly not among tuners—but think how smugly and knowingly you can smirk at the musical world!

HOW DO I TUNE?

Actually, I am suggesting this course so that you will realize how far *real* musical instruments, in the condition they are kept by average owners, deviate from the abstract Platonic concepts of the ivory-tower theorists. I hope it will be a salutary shock! Even on philosophical grounds, the idea of a Perfect Amount of Imperfection is questionable.

Also, it may keep you from knocking yourself out trying to tune better than the professionals can; and make you realize that instruments go out of tune, so that by a month or two afterwards, ultraprecision or the effort to attain it has gone down the drain.

These deviations and instabilities of pitch, so offensive to the abstract intellect, are tolerated to an amazing degree in the actual performance of music. Consequently, only certain styles of composition are going to tolerate great precision—which in turn will profoundly affect one's choice of tuning--systems and instruments.

These considerations save us a lot of time and trouble: all the routines we don't have to construct because they are not worth using. Masochists must look elsewhere.

With that out of the way, let's do something about the 24- or quartertone system. Since the Circle of Twelve Fifths is closed, you can't have a Circle of Twenty-Four Equally-tempered Fifths. So, the simplest way is to get a fork or other standard for either a quartertone higher or lower than 440 Hz and do the 12-tone routine all over again at this other pitch: you simply pretend that A-semisharp 452.9 or A-semiflat 427.5 Hz is A, and take it from there. Voilà!

What if you don't have such a fork, or it would take too many months to get it?

According to some tuners I questioned a while ago, they interpolate the quartertones by ear—which of course means by guess and by gosh. I think I have a better way: if the instrument being tuned is in a place having ordinary power-lines, you can adjust a radio or amplifier or the like to hum at double the power-line frequency. Virtually everywhere in the U.S., the a.c. power is 60 Hz, so the double-frequency or octave higher from a fullwave rectifier, buzzer, transformer hum, etc. is 120 Hz, while the quartertone pitch B-semiflat is 119.956 Hz, flatter than it by less than one beat in 20 seconds, so we can safely ignore that.

So all you would have to do is to come up an octave to 240 Hz, call that B-semiflat, and enter your twelve-tone routine at B-flat.

For the 50-Hz power lines in other parts of the world, you would take the 100-Hz hum as G-semisharp, come up one or two octaves as your 12-tone routine demands, and enter the routine at G or A-flat as you please.

For reasons which you should be able to figure out for yourself, the 100 Hz is not as accurate a quartertone as the 120 Hz, and you might want to make the G-semisharp one beat per second (almost) sharper.

Tuning the 24-tone system by ear is frustrating in a way in which the 19, 22, and 31 systems are not: few people are familiar with the intervals which contain odd numbers of quartertones and link the two circles of 12 fifths each.

If you have not heard the intervals whose just ratios are 4:7 and 8:11, which I call the subminor seventh and the semiaugmented fourth respectively, no amount of written description by me is going to give you what they sound like or what they feel like!

How the blue blazes, then, can I expect you to look up the beat-rates for them in my 24-tone beat-table, and do a good job of tuning the quartertone imitations of those two intervals? (The imitation of the 8:11 is excellent; that of the 4:7 is kinda punk.)

So if you are not prejudiced against electric guitars with quartertone frets, have one made up (a simple matter of intercalation, with my fretting table) and become familiar with all the quartertone intervals. Then you should be able to tune other instruments to quartertones.

Besides guitars, harpsichords, polyphonic synthesizers, certain electronic organs, and psalteries are far superior to pianos for the quartertone system. I really would prefer that you do not use pianos for 24-tone, although they are quite satisfactory for 17- and 19-tone.

Without further ado, let's go on to the19-tone system. This is *easier to tune* than the 12-tone, at least for me; and I have had hundreds more 12--system tunings than 19. You might even learn it first, if you can get a suitable instrument built.

If all you intend to tune is 12 tones out of the 19 on a twelve-tone instrument such as a single ordinary piano, I don't want to help you! You'd be more of a roadblock than a trailblazer. That's sabotage, not progress!

That is to say, when I say that the piano will work in 19-tone, I mean two pianos with the five tones F♯ C♯ G♯ D♯ A♯ in common and seven naturals on the first piano and seven flats on the second. This disposition was used by M. Joel Mandelbaum in the compositions for his 1961 Thesis.

This disposition of two 12-tone keyboards should work for other instruments, such as harpsichords, electronic organs, harmoniums, or synthesizers.

Special keyboards for 19-tone have been invented, and it fits well on many 'generalized' keyboards.

How do I Tune?

On synthesizers and some kinds of organs, the cheapest and quickest way is merely to map the 19 tones onto the conventional 12-tone keyboard, so that 3 octaves of 12 give you not quite 2 of 19, or 5 octaves of 12 give you just over 3 octaves of 19 (this is what I do on my specially-built electronic organ).

This is impossible on master-oscillator-and--divider-type organs, or keyboard stringed instruments, for obvious reasons. But where it is applicable, it will save you agonizing waiting and grieving over unfulfilled dreams.

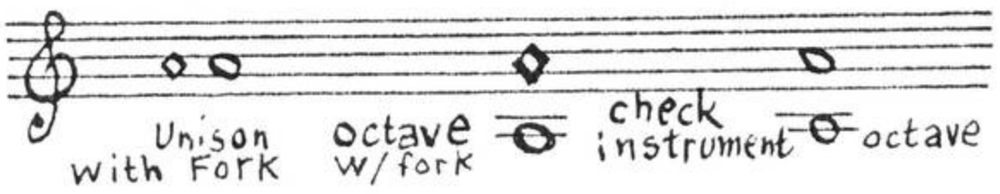
Most master-oscillator-and-divider organs would have the reserve power to run 7 additional divider chains with their oscillators, and while rebuilding the organ, some kind of 19-tone keyboard could be getting made. This, naturally, is an ambitious undertaking, but some of you might be intrepid enough.

I have done enough composing and performing in the 19-tone temperament on different instruments that I am confident the system is worth it, otherwise I wouldn't bother with this information now being dispensed. 19-tone has a peculiar mood and personality and melodic and harmonic properties that no other system has.

A few decades ago, Augusto Novaro of Mexico constructed what he called *cajas de afinación*, —let us say 'tuning-boxes'—which looked like autoharps or zithers, but really were psalteries. This is still a very practical idea, and probably less expensive than Harry Partch's *Harmonic Canons*.

You will see from my 19-tone Beat Table that the minor thirds and major sixths take as much as 17 seconds to go through just *one* beat. One beat in 5 seconds would be far too slow to count. The fraction-of-a-cent error is way beyond the limits of hearing or the stability of tuning of most instruments, so forget it, and consider all major sixths and minor thirds as perfect, just, and beatless. 19 minor thirds in a chain fail to close the Circle by only 3 cents!

19 Fifths of Francisco Salinas' 1/3-comma temperament miss closing the Circle by 0.94 *of* a cent! This is indistinguishable from 19-tone Equal. So, our routine might as well be: Tune Fifths and Fourths as shown in the beat-table, three in a row to get to the next major sixth or minor third, then check each such minor third or major sixth as you go: starting-tone against 3rd link in the series, 1st Fifth-up against the 4th-tuned tone, and so on: 2nd against 5th, 3rd against 6th,&c. Starting from A at 440 Hz, it might be written:





If you are experienced in 12-tone tuning, this or any other 19-tone routine may confuse you. Nothing much I can do about that, except what I said above about getting a 19-tone electric guitar and becoming familiar with the new intervals and some of the chords, just as I advised for the quartertone system.

I can forewarn you of difficulties: the major thirds are *flat*—repeat that to yourself, please. the major thirds are half as flat as the twelve--tone major thirds are sharp. Memorize that!

The fifths are almost the same flatness as the major thirds, *in cents, that is;* but that does not mean they will *beat* at the same rate, as you can see from the table. It does mean that the minor thirds come out almost perfect.

The beat-rates are easier to count than those for 12-tone. This is a great blessing.

The twelvically-indoctrinated tuner will feel peculiar when he gets to the twelfth note to be tuned from the starting-point and it is *not* the same as the starting-tone at all! Seven more

HOW DO I TUNE?

to go. The mere name-change from E-sharp to F-*flat* or B-sharp to C-*flat* will croggle him.

The problems are mostly psychological, matters of *attitude*, not matters of fact. How the tones are represented by *nomenclature* or *notation* is trivial; such questions should not be allowed to discourage and stop people from *hearing the effects of the new systems*. Remember: the critics of such systems as 19-tone are almost all those who have never heard them!

Since the whole affair is psychological, rather than a matter of moral or physical principles, you won't master the non-12 tuning routines if you are prejudiced against them (i.e., have the wrong attitude) or if you don't have good *auditory memory-images of the system in question*. Happily, there is a quick and inexpensive way now to get these: Have a guitar fretted to the system.

If, before you try to tune the intervals, you already know how they should sound, you will make fewer mistakes in the routine. If, on the other hand, you mentally measure everything against 12-tone, it will seem wrong wrong wrong, and you will resent every step and do it badly so that you can tell everybody how horrid it is.

I very strongly suspect that the reason there has been so little done about certain systems is that no tuning-routines were ever made up or published for them. I wonder how many people believe that there is no possible routine other than that particular 12-tone one they have learnt or observed.

As with twelve-tone, there are certain chords which can be used to check the 19-tone tuning after some of the octaves are tuned (or right away, in the case of a locked-octaves organ).

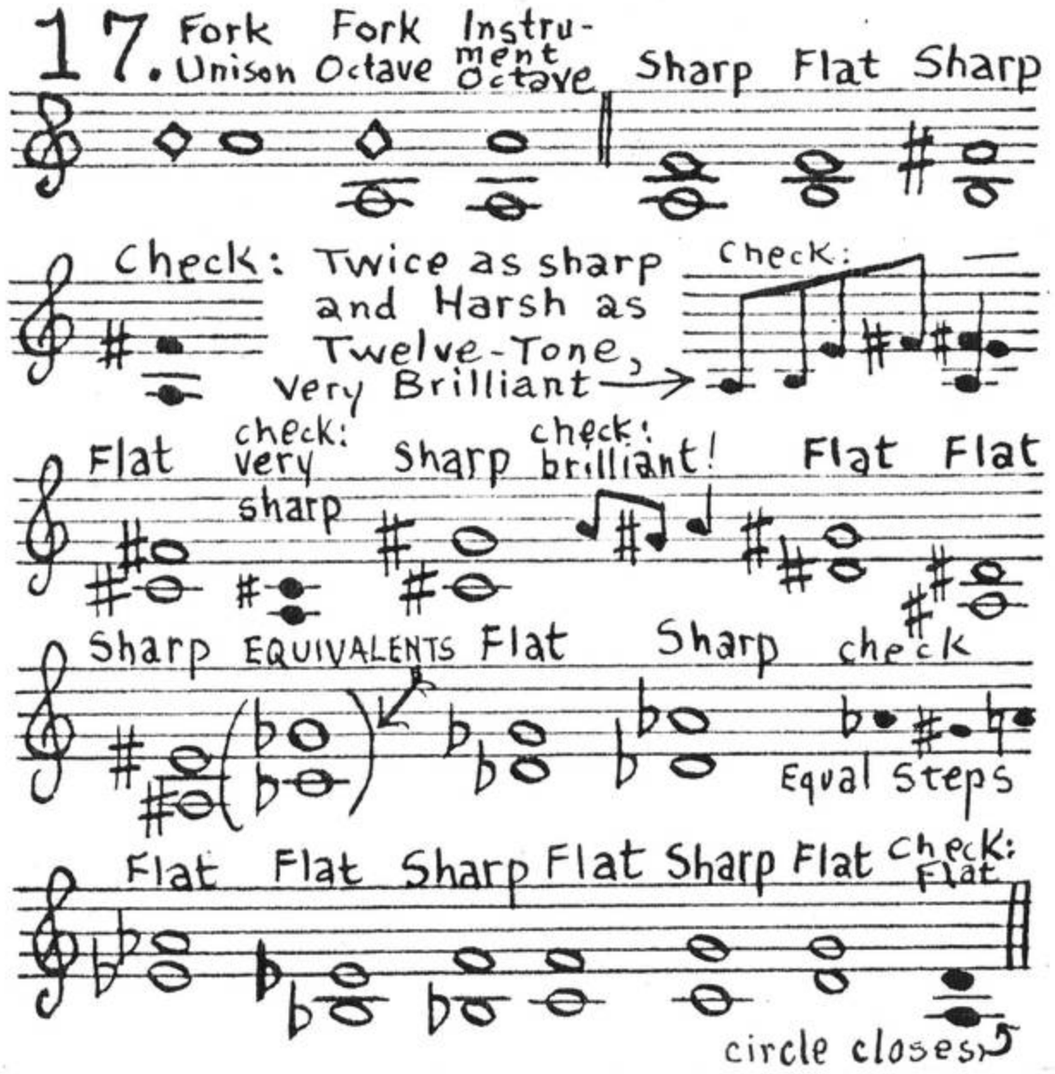
In the case of other organs and other instruments, the 19-tone system allows stretching of the octaves to a much greater extent than 12 does. Special routines and tables will be provided in the future.



The preceding only scratches the surface, of course. The important point is *not* to check with any music written by someone who never heard the nineteen-tone scale in all his life! That would be egging on all hecklers to come and do their worst, which they gladly will!!

Now we come to the 17-tone system. It will be more difficult to learn and practice the seventeen-tone routine because there are no good checking-intervals along the way. Instead of restful, just-about-perfect minor thirds and major sixths, and passable major thirds and minor sixths, their 17-system counterparts are very harsh and dissonant. This turns the rules of harmony upside-down!

Mind what you are about. The fifths are SHARP *and* the fourths are FLAT.



Note that A-sharp is C-flat and E-sharp is G-flat. Also, A-sharp is sharper than B-flat, or, a sharp is *two* - seventeenths of an octave.

Find a brilliant melody for solo violin or trumpet, and play it in 17-tone. Then try pentatonic melodies and chords-in-fourths.



How do I Tune?

Paradoxically, when the routine is constructed for the 34-tone system, it will make it *easier*, because major and minor thirds and sixths will "bridge" both circles of 17 fifths and thus tie in at many points. That will provide the checks which 17 cannot have.

When we come to the 22-tone system, notation and nomenclature pose more problems than the routine for tuning it. I have tuned this system in a number of ways which again are difficult to describe but if anything easier than ordinary twelve-tone routines. The thirds and fifths and septimal intervals interlock in a mutually-checking manner.

For instance, one may proceed much as one would to tune a just major scale on the starting-tone. Then add the 7th harmonic as imitated in 22-tone, then a chain of three major thirds to get the "quartertone" between major seventh and octave. This done, there are a number of ways of filling in the remaining tones. That is, it is not necessary to doggedly struggle through the very long circle of 22 fifths. The beat-table works just as well, whatever the chosen routine.

The errors of the 22-tone major third and fifth *add* instead of cancelling out as they do in 19; therefore the minor third is considerably sharp, but the major seventh and diatonic semitone are within 3 cents of just values. This alone dictates a different kind of tuning-procedure from that for 12, 17, or 19.

The approximations available to several intervals unfamiliar to most tuners, provide additional mutual-checking-points with which to sew the twenty-two-tone fabric together. It will help considerably if you can sound and become familiar with the subminor third whose just ratio is 6:7, the subminor triad whose just ratio is 6:7:9, the supermajor third whose just ratio is 7:9 and which is approximated only one cent away by 22-tone; and the semi-augmented fourth whose just ratio is 8:11.

We now are at a point where cold reasoning and dry theory and argument from traditional 12-tone principles are worse than useless. There is no way to put down on paper what the 22-tone equal temperament *sounds like;* and most emphatically no way to describe the 22-tone *mood* other than to play some worthwhile music on an instrument so tuned.

There isn't space in this issue to set out routines in full. Why not get a guitar refretted in 22-tone, so you can become familiar with the effect of the system, and its intervals?

Xenharmonic Bulletin No. 7 contained fretting-tables for 17, 19, 22, 24, 31, and 34 tones per octave. This means that there is no longer any pressure on anyone to learn how to use routines or set temperaments by ear. We will nevertheless give routines for 22-tone, 34-tone, meantone, 31-tone, and some stretched-octave systems in future Bulletins for those who might like to try them, or who prefer to check tunings arrived at by other means.

With all of the routines, the commonest mistake is to tune an interval to the right number of beats per second, but on the *wrong side* of beatlessness: sharp instead of flat, or flat instead of sharp. Remember to tune *every* interval just, before tempering it.

The peculiarities of piano strings are such as to make the partial tones sharper than the true harmonics, so that stretched-octave tuning has become a custom. The stretch itself is stretched, at both ends of the keyboard. Now, the 19- and 31-tone systems could do with stretched octaves, regardless of what instrument is to be tuned. Applying these systems to pianos will compound the situation, obviously.

On the contrary, such systems as 17, 34, and 22 might do better with *shrunken* octaves. This will sound 'queer' on pianos, and personal prejudices should be taken into account.

Organs and harpsichords are ordinarily not stretched-octave-tuned, or only slightly; but in the cases of 19- and 31-tone (meantone also) this question must be reëxamined.

You will notice that I have not said much about a very large family of tuning-routines which consists of unequal temperaments, variations on 12-tone-equal, variations on 12--out of-meantone, muzzled wolves, schemes with different-sized fifths, with a few just fifths along with other tempered ones, etc. This has been deliberate, intentional, and on purpose!

I do not want to encourage the tuning of systems which have only twelve different pitches per octave, no matter how these may be spaced, because none of them offer today's composers anything different enough from 12-tone equal, because anything different enough will need more notes in order to be *complete.* That is what xenharmonics is all about!

As a composer I will not accept the restrictions of a bygone age, arising from mechanical and financial problems which now have disappeared. Laudable though the attempt to get old keyboard music performed in a better or more authentic intonation may be, this is not the central objective of the *Xenharmonic Bulletin.*

That is to say, xenharmonics is not the re-exploration of forgotten bypaths, but the blazing of new trails! Nor is it the search for exotica via ethnomusicology, the mere copying of distant cultures in the hope it won't sound like warmed-over Rimskii-Korsakov or Debussy.

For Pitch a′ = 440 Hz ; 17-tone c′ = 258.972 Hz

17-TONE EQUAL TEMPERAMENT: BEATS PER SECOND FOR FIFTHS AND FOURTHS
Based upon frequency table compiled by Ervin M. Wilson

FIFTHS (sharp)			FOURTHS (flat)		
c	g	0.88	c	f	1.17
d♭	a♭	0.92	d♭	g♭	1.22
c♯	g♯	0.96	c♯	f♯	1.27
d	a	1.00	d	g	1.33
e♭	b♭	1.04	e♭	a♭	1.38
d♯	a♯	1.08	d♯	g♯	1.44
e	b	1.13	e	a	1.50
f	c′	1.17	f	b♭	1.56
g♭	d′♭	1.22	e♯/g♭	a♯	1.63
f♯	c′♯	1.27	f♯	b	1.70
g	d′	1.33	g	c′	1.77
a♭	e′♭	1.38	a♭	d′♭	1.84
g♯	d′♯	1.44	g♯	c′♯	1.91
a	e′	1.50	a	d′	2.00
b♭	f′	1.56	b♭	e′♭	2.08
a♯	e′♯/g′♭	1.63	a♯	d′♯	2.17
b	f′♯	1.70	b	e′	2.25
c′	g′	1.77	c′	f′	2.35
d′♭	a′♭	1.84	d′♭	g′♭	2.44
c′♯	g′♯	1.91	c′♯	f′♯	2.54
d′	a′	2.00	d′	g′	2.65
e′♭	b′♭	2.08	e′♭	a′♭	2.76
d′♯	a′♯	2.17	d′♯	g′♯	2.88
e′	b′	2.25	e′	a′	3.00
f′	c″	2.35	f′	b′♭	3.12
g′♭	d″♭	2.44	e′♯/g′♭	a′♯	3.25
f′♯	c″♯	2.54	f′♯	b′	3.39
g′	d″	2.65	g′	c″	3.53
a′♭	e″♭	2.76	a′♭	d″♭	3.68
g′♯	d″♯	2.88	g′♯	c″♯	3.83
a′	e″	3.00	a′	d″	3.99
b′♭	f″	3.12	b′♭	e″♭	4.16
a′♯·e″♯/g″♭		3.25	a′♯	d″♯	4.33
b′	f″♯	3.39	b′	e″	4.50
c″	g″	3.53	c″	f″	4.70

REMARKS: There is no point in giving beat-rates for other intervals in this 17-tone table, because they are so fast as to be useless for tuning purposes.

For instance, the major third in the 17--tone temperament on Middle C, c′-- e′ , is so sharp that it beats 28.14 times per second, which is almost three times the beat-rate of the corresponding 12--tone-tempered major third (10.38 / second).

The major sixth in 17-tone temperament on Middle C, c′-- a′, has a beat-rate of 25.14 per second, obviously much too fast to count.

Otherwise stated, the thirds and sixths of 17-tone temperament are dissonances asking for resolution.

— *Ivor Darreg*

Frequency and Beat Tables for the 34-tone equal temperament, which has harmonious thirds and sixths, will be available.

FREQUENCY-TABLE FOR 19-TONE EQUAL TEMPERAMENT, A=440 Hz

0 C	1 C#	2 Db	3 D	4 D#
19 16.501	17.115	17.750	18.410	19.096
33.003	34.229	35.501	36.820	38.188
66.006	68.458	71.002	73.640	76.376
132.011	136.916	142.003	147.280	152.752
264.023	273.832	284.007	294.559	305.503
528.045	547.665	568.013	589.118	611.006
1056.090	1095.329	1136.026	1178.236	1222.013
2112.181	2190.659	2272.053	2356.471	2444.026
4224.362	4381.318	4544.106	4712.942	4888.052
8448.723	8762.635	9088.211	9425.885	9776.103

5 Eb	6 E	7 E#/Fb	8 F	9 F#
19.803	20.539	21.302	22.094	22.915
39.607	41.078	42.605	44.188	45.829
79.214	82.157	85.209	88.375	91.659
158.427	164.313	170.419	176.750	183.318
316.854	328.627	340.837	353.501	366.635
633.709	657.254	681.674	707.002	733.271
1267.417	1314.508	1363.349	1414.003	1466.541
2534.835	2629.016	2726.698	2828.007	2933.082
5069.669	5258.032	5453.395	5656.013	5866.164
10139.338	10516.063			

•10 Gb	11 G	12 G#	13 Ab	14 A
23.766	24.649	25.565	26.515	27.500
47.532	49.298	51.130	53.030	55.000
95.064	98.597	102.260	106.059	110.000
190.129	197.193	204.520	212.119	220.000
380.258	394.386	409.040	424.237	440.000
760.515	788.772	818.079	848.475	880.000
1521.031	1577.544	1636.158	1696.950	1760.000
3042.061	3155.089	3272.317	3393.899	3520.000
6084.123	6310.177	6554.634	6787.799	7040.000

15 A#	16 Bb	17 B	18 B#/Cb
28.522	29.582	30.681	31.821
57.044	59.163	61.361	63.641
114.087	118.326	122.722	127.282
228.174	236.652	245.445	254.564
456.348	473.304	490.890	509.128
912.696	946.608	981.779	1018.257
1825.393	1893.216	1963.558	2036.514
3650.786	3786.432	3927.116	4073.027
7301.571	7572.865	7854.232	8146.054

Values for the 19th root of 2 and its powers from Equal Temperament Table by John H. Chalmers, Jr
The range of this table represents the compass of large organs, 9 octaves and a major third.

(For Pitch A'=440 Hz, 19-tone C'=264,023)

19-TONE EQUAL TEMPERAMENT: BEATS PER SECOND FOR CERTAIN INTERVALS

FIFTHS (flat)		FOURTHS (sharp)		MAJOR THIRDS (flat)		MINOR SIXTHS (sharp)	
c — g	1.65	c — f	2.21	G — B	2.09	G — e♭	3.36
c♯ — g♯	1.71	c♯ — f♯	2.29	G♯ — B♯	2.17	G♯ — e	3.49
d♭ — a♭	1.77	d♭ — g♭	2.37	A♭ — c	2.25	A♭ — f♭	3.62
d — a	1.84	d — g	2.46	A — c♯	2.34	A — f	3.75
d♯ — a♯	1.91	d♯ — g♯	2.55	(A♯ — cx B𝄫 — d♭)	2.42	A♯ — f♯	3.89
e♭ — b♭	1.98	e♭ — a♭	2.65			B♭ — g♭	4.04
e — b	2.05	e — a	2.75	B♭ — d	2.51	B — g	4.19
(e♯ — b♯ f♭ — c'♭)	2.13	(e♯ — a♯ f♭ — b𝄫)	2.85	B — d♯	2.61	B♯ — g♯	4.34
				c♭ — e♭	2.70	c — a♭	4.50
f — c'	2.21	f — b♭	2.95	c — e	2.80	c♯ — a	4.67
f♯ — c'♯	2.29	f♯ — b	3.06	c♯ — e♯	2.91	(cx — a♯ d♭ — b𝄫)	4.84
g♭ — d'♭	2.37	(g♭ — c♭ fx — b♯)	3.18	d♭ — f	3.01		
g — d'	2.46			d — f♯	3.13	d — b♭	5.02
g♯ — d'♯	2.55	g — c'	3.30	(d♯ — fx e𝄫 — g♭)	3.24	d♯ — b	5.21
a♭ — e'♭	2.65	g♯ — c'♯	3.42			e♭ — c'♭	5.40
a — e'	2.75	a♭ — d'♭	3.55	e♭ — g	3.36	e — c'	5.61
(a♯ — e'♯ b𝄫 — f'♭)	2.85	a — d'	3.68	e — g♯	3.49	e♯ — c♯	5.81
		a♯ — d'♯	3.81	f♭ — a♭	3.62	f — d'♭	6.03
b♭ — f'	2.95	b♭ — e'♭	3.96	f — a	3.75	f♯ — d'	6.25
b — f'♯	3.06	b — e'	4.10	f♯ — a♯	3.89	(fx — d♯ g♭ — e𝄫)	6.49
(c'♭ — g' b♯ — fx)	3.18	(b♯ — e♯ c'♭ — f'♭)	4.25	g♭ — b♭	4.04		
				g — b	4.19	g — e'♭	6.73
c' — g'	3.30	c' — f'	4.41	g♯ — b♯	4.34	g♯ — e'	6.98
c'♯ — g'♯	3.42	c'♯ — f'♯	4.58	a♭ — c'	4.50	a♭ — f'♭	7.24
d'♭ — a'♭	3.55	d'♭ — g'♭	4.75	a — c'♯	4.67	a — f'	7.50
d' — a'	3.68	d' — g'	4.92	(a♯ — c'x b𝄫 — d'♭)	4.84	a♯ — f'♯	7.78
d'♯ — a'♯	3.81	d'♯ — g'♯	5.11			b♭ — g'♭	8.07
e'♭ — b'♭	3.96	e'♭ — a'♭	5.30	b♭ — d'	5.02	b — g'	8.38
e' — b'	4.10	e' — a'	5.49	b — d♯	5.21	b♯ — g'♯	8.68
(f'♭ — c"♭ e'♯ — b'♯)	4.25	(e'♯ — a'♯ f'♭ — b'𝄫)	5.70	c'♭ — e'♭	5.40	c' — a'♭	9.01
				c' — e'	5.61	c'♯ — a'	9.34
f' — c"	4.41	f' — b'♭	5.91	c'♯ — e'♯	5.81	(d'♭ — b'𝄫 cx — a♯)	9.69
f'♯ — c"♯	4.58	f'♯ — b'	6.13	d'♭ — f'	6.03		
g'♭ — d"♭	4.75	(f'x — b'♯ g'♭ — c"♭)	6.35	d' — f'♯	6.25	d' — b'♭	10.05
g' — d"	4.92			(d'♯ — f'x e'𝄫 — g'♭)	6.49	d'♯ — b'	10.42
g'♯ — d"♯	5.11	g' — c"	6.59			e'♭ — c"♭	10.81
a'♭ — e"♭	5.30	g'♯ — c"♯	6.84	e'♭ — g'	6.73	e' — c"	11.21
a' — e"	5.49	a'♭ — d"♭	7.09	e' — g♯	6.98		
(a'♯ — e"♯ b'𝄫 — f"♭)	5.70	a' — d"	7.35	f'♭ — a'♭	7.24		
		a'♯ — d"♯	7.63	f' — a'	7.50		
b'♭ — f"	5.91	b'♭ — e"♭	7.91	f'♯ — a'♯	7.78		
b' — f"♯	6.13	b' — e"	8.20	g'♭ — b'♭	8.07		
(c"♭ — g"♭ b'♯ — f"x)	6.35	(b' — e"♯ c"♭ — f"♭)	8.51	g' — b'	8.38		
				g'♯ — b'♯	8.68		
c" — g"	6.59	c" — f"	8.82				

Nineteen-tone-system name-equivalents are bracketed. Small c means Tenor C second space bass staff 132.011 Hz in 19-tone. c' means Middle C.

(For Pitch a'=440 Hz; 19-tone c' = 264.023 Hz)

19-TONE EQUAL TEMPERAMENT: BEATS PER SECOND FOR CERTAIN INTERVALS, CONTINUED

MINOR THIRDS (sharp)		minor thirds (continued)	
c — e♭	0.068	a′ — c″	.226
c♯ — e	.070	a′♯ — c″♯	.235
d♭ — f♭	.073	b′♭ — d″♭	.242
d — f	.075	b′ — d″	.251
d♯ — f♯	.078	(b′♯ — d″♯ / c″♭ — e″𝄫)	.263
e♭ — g♭	.081	c″ — e″♭	.273
e — g	.084	c″♯ — e″	.281
e♯ — g♯	.087	d″♭ — f″♭	.293
f — a♭	.091	d″ — f″	.300
f♯ — a	.094	d″♯ — f″♯	.314
(f𝄪 — a♯ / g♭ — b𝄫)	.098	e″♭ — g″♭	.323
g — b𝄫	.102	e″ — g″	.336
g♯ — b	.105	e″♯ — g″♯	.350
a♭ — c♭	.109	f″ — a″♭	.364
a — c′	.113	f″♯ — a″	.378
a♯ — c′♯	.117	(f″𝄪 — a″♯ / g″♭ — b″𝄫)	.391
b𝄫 — d′♭	.121	g″ — b″♭	.408
b — d′	.126	g″♯ — b″	.421
b♯ — d′♯	.131	a″♭ — c‴♭	.435
c′ — e′♭	.136	a″ — c‴	.452
c′♯ — e′	.141	a″♯ — c‴♯	.470
d′♭ — f′♭	.146	b″♭ — d‴♭	.484
d′ — f′	.150	b″ — d‴	.503
d′♯ — f′♯	.157	(b″♯ — d‴♯ / c‴♭ — e‴𝄫)	.525
e′♭ — g′♭	.162	c‴ — e‴♭	.546
e′ — g′	.168		
e′♯ — g′♯	.175		
f′ — a′♭	.182		
f′♯ — a′	.189		
(f′𝄪 — a′♯ / g′♭ — b′𝄫)	.195		
g′ — b′♭	.204		
g′♯ — b′	.210		
a′♭ — c″♭	.218		

MAJOR SIXTHS (flat)		major sixths (continued)	
c — a	0.057	g′♯ — e″♯	.175
c♯ — a♯	.059	a′♭ — f″	.182
d♭ — b𝄫	.061	a′ — f″♯	.189
d — b	.063	(a′♯ — f″𝄪 / b′𝄫 — g″♭)	.195
(d♯ — b♯ / e𝄫 — c′♭)	.066	b′♭ — g″	.204
e♭ — c′	.068	b′ — g″♯	.210
e — c′♯	.070	c″♭ — a″♭	.218
(e♯ — c′𝄪 / f♭ — d′♭)	.073	c″ — a″	.226
f — d′	.075	c″♯ — a″♯	.235
f♯ — d′♯	.078	d″♭ — b″♭	.242
g♭ — e′♭	.081	d″ — b″	.251
g — e′	.084	(d″♯ — b″♯ / e″𝄫 — c‴♭)	.263
g♯ — e′♯	.087	e″♭ — c‴	.273
a♭ — f′	.091	e″ — c‴♯	.281
a — f′♯	.094	f″♭ — d‴♭	.293
(a♯ — f′𝄪 / b𝄫 — g′♭)	.098	f″ — d‴	.300
b𝄫 — g′	.102	f″♯ — d‴♯	.314
b — g′♯	.105	g″♭ — e‴♭	.323
c′♭ — a′♭	.109	g″ — e‴	.336
c′ — a′	.113	g″♯ — e‴♯	.350
c′♯ — a′♯	.117	a″♭ — f‴	.364
d′♭ — b′♭	.121	a″ — f‴♯	.378
d′ — b′	.126	(a″♯ — f‴𝄪 / b″𝄫 — g‴♭)	.391
(d′♯ — b′♯ / e′𝄫 — c″♭)	.131	b″♭ — g‴	.408
e′♭ — c″	.136	b″ — g‴♯	.421
e′ — c″♯	.141	c‴♭ — a‴♭	.435
(e′♯ — c″𝄪 / f′♭ — d″♭)	.146	c‴ — a‴	.452
f′ — d″	.150		
f′♯ — d″♯	.157		
g′♭ — e″♭	.162		
g′ — e″	.168		

SUBMINOR SEVENTHS (flat)		subminor sevenths (continued)	
C — A♯	5.69	G♭ — e	8.20
C♯ — B♭	5.90	G — e♯	8.50
D♭ — B	6.12	G♯ — f	8.82
D♯ — c♭	6.35	A♭ — f♯	9.15
D♯ — c	6.59	A — g♭	9.48
E♭ — c♯	6.83	A♯ — g	9.84
E — d♭	7.08	B♭ — g♯	10.20
E♯ — d	7.35	B — a♭	10.58
F — d♯	7.62	B♯ — a	10.97
F♯ — e♭	7.90	c — a♯	11.38

REMARKS: Beat-rates have been given outside some of the usual tuning octaves to facilitate evaluations and to make them easier to count. The designations of notes and intervals of the nineteen-tone scale used here are NOT intended to imply any recommendations, or disparagements of other nomenclatures. You may call them whatever you like.

TWENTY-TWO-TONE EQUAL TEMPERAMENT: TABLE OF FREQUENCIES
BASED UPON STANDARD PITCH, A⁻ = 440 Hz
Carried through Nine Octaves and a Major Third.

NOTE: The unfamiliar nature of name-synonym-equivalents in the 22-tone system renders it advisable to create an auxiliary set of "plus" and "minus" note--designations, and also to encourage the use of note-numbers (cf. Carrillo's system of number-notation). To place plain unmarked C, the usual reference-point, near its familiar location, "A-minus" (which belongs in the C-major scale), not plain A, has been taken as 440 hertz. — *Ivor Darreg*.

0 (22) C B#-- Ax-	1 Db B#- C+	2 B# Ebb C#- Db+	3 C# D- Ebb+
16.611	17.143	17.692	18.258
33.222	34.286	35.383	36.516
66.445	68.572	70.767	73.032
132.890	137.143	141.533	146.063
265.780	274.287	283.066	292.127
531.559	548.574	566.133	584.254
1063.119	1097.147	1132.265	1168.507
2126.237	2194.295	2264.530	2337.014
4252.475	4388.589	4529.060	4674.029
8504.950	8777.179	9058.121	9348.047

4 D Ebb++ Cx--	5 Eb Bx D#-- Cx-	6 Fb Cx D#- Eb+	7 E- Fb+ Gbb D#
18.842	19.445	20.068	20.710
37.685	38.891	40.136	41.420
75.369	77.782	80.271	82.841
150.639	155.563	160.543	165.682
301.477	311.127	321.086	331.363
602.954	622.254	642.171	662.726
1205.909	1244.508	1284.342	1325.452
2411.818	2489.016	2568.685	2650.904
4823.635	4978.032	5137.370	5301.808
9647.270	9956.063	10274.740	10603.617

8 E F- Fb++ Gbb+	9 F E#-- Gbb++	10 Gb F+ E#- Dx	11 F#- Gb+ E# Abb
21.373	22.057	22.763	23.492
42.746	44.114	45.526	46.984
85.492	88.229	91.053	93.967
170.985	176.458	182.106	187.935
341.969	352.915	364.211	375.869
683.939	705.830	728.423	751.738
1367.878	1411.661	1456.856	1503.477
2735.755	2823.322	2913.692	3006.954
5471.510	5646.644	5827.383	6013.908

(Values for the twenty-second root of two and its powers were supplied by Ervin M. Wilson. This table was recently rechecked against the equal temperament tables of John H. Chalmers, Jr.)

TWENTY-TWO-TONE EQUAL TEMPERAMENT: TABLE OF FREQUENCIES, CONTINUED

12 F# G- Gb++	13 G Abb++ Fx--	14 Ab Ex G#--	15 Ab+ Bbb Fx G#-
24.244	25.020	25.821	26.647
48.488	50.040	51.641	53.294
96.975	100.079	103.282	106.588
193.950	200.158	206.565	213.177
387.900	400.316	413.130	426.353
775.800	800.632	826.259	852.706
1551.601	1601.265	1652.518	1705.413
3103.201	3202.529	3305.037	3410.825
6206.402	6405.059	6610.074	6821.651

16 A- G# Cbb Bbb+	17 A Bbb++	18 Bb A#-- Gx-	19 Bb+ Gx A#- Cb
27.500	28.380	29.289	30.226
55.000	56.760	58.577	60.452
110.000	113.521	117.155	120.904
220.000	227.042	234.309	241.809
440.000	454.084	468.618	483.618
880.000	908.167	937.236	967.235
1760.000	1816.335	1874.472	1934.471
3520.000	3632.669	3748.945	3868.942
7040.000	7265.338	7497.889	7737.884

20 B- A# Dbb Cb+	21 C- B Cb++ Dbb+
31.194	32.192
62.387	64.384
124.774	128.768
249.549	257.536
499.098	515.073
998.195	1030.146
1996.390	2060.291
3992.780	4120.582
7985.560	8241.164

Nine octaves and a major third is the greatest compass of fundamental frequencies likely to be encountered, even on the largest organs, and therefore has been chosen as the limit in this series of tables.

The "plus" and "minus" note-names were suggested by the fact that the unit interval of the system, 1/22 octave, often behaves as a "pseudocomma" despite being more than twice the size of a real comma. Three plusses make one sharp and three minuses make one flat. So it will be neater to write B than to write B#---, even though the latter expression emphasizes that the augmented seventh is a pile of three major thirds. This visual/semantic hassle can be evaded by calling that note No. 21.

(For Pitch a′=440 Hz; 22-Tone c′=265.780 Hz)

22-TONE EQUAL TEMPERAMENT: BEATS PER SECOND FOR CERTAIN INTERVALS

NAME(S) LOWER TONE	NO. FROM C	FIFTHS (sharp)	FOURTHS (flat)		MAJOR THIRDS (flat)	MINOR SIXTHS (sharp)
G	13	1.24	1.65		1.30	2.08
A♭ G#=	14	1.28	1.70		1.34	2.15
A♭+ G#−	15	1.32	1.76		1.38	2.22
A− G#	16	1.36	1.81	*110 Hz*	1.43	2.29
A	17	1.41	1.87		1.47	2.36
B♭	18	1.45	1.93		1.52	2.44
B♭+ A#−	19	1.50	1.99		1.57	2.51
B− A#	20	1.55	2.05		1.62	2.60
B c−	21	1.60	2.12		1.67	2.68
c	0(22)	1.65	2.19		1.72	2.76
c+ d♭	1	1.70	2.26		1.78	2.85
d♭+ c#−	2	1.75	2.33		1.84	2.94
d− c#	3	1.81	2.40		1.89	3.04
d	4	1.87	2.48		1.95	3.14
e♭	5	1.93	2.56		2.02	3.24
e♭+ d#−	6	1.99	2.64		2.08	3.34
e− d#	7	2.05	2.73		2.15	3.45
e	8	2.12	2.81		2.22	3.56
f	9	2.19	2.90		2.29	3.67
f+ g♭	10	2.26	3.00		2.36	3.79
g♭+ f#−	11	2.33	3.09		2.44	3.91
g− f#	12	2.40	3.19		2.51	4.03
g	13	2.48	3.29		2.60	4.16
a♭ g#=	14	2.56	3.40		2.68	4.30
a♭+ g#−	15	2.64	3.51		2.76	4.43
a−	16	2.73	3.62	*220Hz*	2.85	4.58
a	17	2.81	3.74		2.94	4.72
b♭	18	2.90	3.86		3.04	4.87
b♭+ a#−	19	3.00	3.98		3.14	5.03
b− a#	20	3.09	4.11		3.24	5.19
b c′−	21	3.19	4.24		3.34	5.36
c′	0(22)	3.29	4.37		3.45	5.53
c′+ d′♭	1	3.40	4.51		3.56	5.71
d′♭+	2	3.51	4.66		3.67	5.89
d′− c′#	3	3.62	4.81		3.79	6.08
d′	4	3.74	4.96		3.91	6.27
e′♭	5	3.86	5.12		4.03	6.47
e′♭+ d′#−	6	3.98	5.28		4.16	6.68
e′− d′#	7	4.11	5.45		4.30	6.89
e′	8	4.24	5.63		4.43	7·11
f′	9	4.37	5.81		4.58	7.34
f′+ g′♭	10	4.51	5.99		4.72	7.58
f′#− g′♭+	11	4.66	6.18		4.87	7.82
g′− f′#	12	4.81	6.38		5.03	8.07
g′	13	4.96	6.59		5.19	8.33
a′♭	14	5.12	6.80		5.36	8.59
a′♭+	15	5.28	7.02		5.53	8.87
a′−	16	5.46	7.24	*440Hz*	5.70	9.16

NOTE: The names given above for the degrees of the 22-tone tempered scale are unofficial expedients. Use of numbers instead is worth considering.

(For Pitch a'- = 440 Hz; 22-Tone c' = 265.780 Hz)

22-TONE EQUAL TEMPERAMENT: BEATS PER SECOND FOR CERTAIN INTERVALS, continued

NAME(S) LOWER TONE		NO.. FROM C	MINOR THIRDS (sharp)	MAJOR SIXTHS (flat)	SUBMINOR SEVENTHS (sharp)	SEMIAUGMENTED FOURTHS (flat)
C		0(22)	2.69	2.23	3.50	2.47
C+	Db	1	2.73	2.30	3.62	2.55
Db	C#-	2	2.86	2.37	3.73	2.63
D-	C#	3	2.95	2.45	3.85	2.72
D		4	3.05	2.52	3.98	2.80
Eb		5	3.15	2.60	4.10	2.89
Eb+	D#-	6	3.25	2.69	4.24	2.99
E-	D#	7	3.35	2.73	4.37	3.08
E		8	3.46	2.86	4.51	3.18
F		9	3.57	2.95	4.65	3.28
F+	Gb	10	3.68	3.05	4.80	3.39
Gb+	F#-	11	3.76	3.15	4.96	3.49
G-	F#	12	3.92	3.25	5.12	3.61
G		13	4.05	3.35	5.28	3.72
Ab	G#=	14	4.18	3.46	5.45	3.84
Ab+	G#-	15	4.31	3.57	5.62	3.96
A-	G#	16	4.45	3.68	5.80	4.09
A		17	4.59	3.76	5.99	4.22
Bb		18	4.74	3.92	6.18	4.36
Bb+	A#-	19	4.89	4.05	6.38	4.50
B-	A#	20	5.04	4.18	6.58	4.64
B	c-	21	5.21	4.31	6.79	4.79
c		0(22)	5.38	4.45	7.01	4.94
c+	db	1	5.45	4.59	7.23	5.10
db+	c#=	2	5.72	4.74	7.46	5.26
d-	c#	3	5.91	4.89	7.70	5.43
d		4	6.10	5.04	7.95	5.61
eb	d#=	5	6.29	5.21	8.20	5.78
eb+	d#-	6	6.49	5.38	8.47	5.97
e-	d#	7	6.69	5.45	8.74	6.16
e		8	6.92	5.72	9.02	6.36
f		9	7.14	5.91	9.30	6.56
f+	gb	10	7.36	6.10	9.60	6.77
gb+	f#-	11	7.52	6.29	9.91	6.99
g-	f#	12	7.84	6.49	10.23	7.21
g		13	8.10	6.69	10.55	7.44
ab	g#=	14	8.36	6.92	10.89	7.68
ab+	g#-	15	8.62	7.14	11.24	7.93
a-	g#	16	8.90	7.36	11.60	8.18
a		17	9.18	7.52	11.97	8.44
bb	a#=	18	9.48	7.84	12.36	8.71
bb+	a#-	19	9.78	8.10	12.75	8.99
b-	a#	20	10.09	8.36	13.16	9.28
b	c'-	21	10.42	8.62	13.58	9.57
c'		0(22)	10.75	8.90	14.01	9.88

The pitch we designate as "A-minus" was chosen as 440 Hz rather than the one designated plain "A" (3 Fifths from C) to bring C (unmarked) within a more conventional limit. This page of the table was "moved down" to make the beats slow enough for practicable counting or measuring.

TABLE OF FREQUENCIES BASED UPON STANDARD PITCH, A = 440 HERTZ

FOR THE TWENTY-FOUR-TONE (QUARTERTONE) EQUAL TEMPERAMENT

WHICH INCLUDES THE CUSTOMARY TWELVE-TONE EQUAL TEMPERAMENT

CARRIED THROUGH NINE OCTAVES AND A MAJOR THIRD

(Notation used here for quartertones is the Couper modification of the Vyshnegradski System.)

0(24) C B# Dbb	1 C♯ D𝄲	2 C# Db	3 C# D𝄳
16.352	16.831	17.324	17.832
32.703	33.661	34.648	35.663
65.406	67.323	69.648	71.326
130.813	134.646	138.591	142.652
261.626	269.292	277.183	285.305
523.251	538.584	554.365	570.610
1046.502	1077.168	1108.731	1141.219
2093.005	2154.335	2217.461	2282.438
4186.009	4308.668	4454.924	4564.877
8372.018	8617.340	8869.848	9129.754

4 D Cx Eb	5 D♯ E𝄲	6 D# Eb	7 D# E𝄳
18.354	18.892	19.445	20.015
36.708	37.784	38.891	40.030
73.416	75.567	77.782	80.061
146.832	151.135	155.563	160.122
293.665	302.270	311.127	320.244
587.330	604.540	622.254	640.487
1174.659	1209.080	1244.508	1280.975
2349.318	2418.159	2489.016	2561.949
4698.636	4836.318	4978.032	5123.899
9397.272	9673.636	9956.063	10247.797

8 E Dx Fb	9 E♯ F𝄳	10 F E# Gbb	11 F♯ G𝄲
20.601	21.205	21.827	22.466
41.203	42.411	43.654	44.933
82.407	84.822	87.307	89.865
164.814	169.643	174.614	179.731
329.628	339.286	349.228	359.461
659.255	678.573	698.456	718.923
1318.510	1357.146	1396.913	1437.845
2637.021	2714.291	2793.826	2875.690
5274.040	5428.583	5587.652	5751.381
10548.080			

TABLE OF FREQUENCIES BASED ON A = 440 Hz FOR THE TWENTY-FOUR-TONE (QUARTERTONE) AND TWELVE-TONE EQUAL TEMPERAMENTS, CONTINUED

12 F # G♭	13 F♯ G𝄳	14 G Fx A𝄫	15 G𝄲 A𝄬
23.124	23.802	24.499	25.218
46.249	47.605	48.999	50.435
92.499	95.209	97.999	100.870
184.997	190.418	195.998	201.741
369.994	380.836	391.995	403.482
739.989	761.672	783.991	806.964
1479.978	1523.344	1567.982	1613.927
2959.955	3046.688	3135.964	3227.854
5919.910	6093.377	6271.928	6455.708

16 G♯ A♭	17 G♯ A𝄳	18 A Gx B𝄫	19 A𝄲 B𝄬
25.956	26.717	27.500	28.306
51.913	53.434	55.000	56.612
103.826	106.869	110.000	113.223
207.652	213.737	220.000	226.446
415.305	427.474	440.000	452.893
830.609	854.948	880.000	905.786
1661.219	1709.896	1760.000	1811.572
3322.438	3419.793	3520.000	3623.143
6644.876	6839.586	7040.000	7246.286

20 A♯ B♭	21 A♯ B𝄳	22 B Ax C♭	23 B𝄲 C𝄳
29.135	29.989	30.868	31.772
58.270	59.978	61.735	63.544
116.541	119.956	123.471	127.089
233.082	239.912	246.942	254.178
466.164	479.824	493.883	508.355
932.327	959.647	987.767	1016.711
1864.655	1919.294	1975.533	2033.421
3729.310	3838.588	3951.066	4066.843
7458.619	7677.176	7902.132	8133.685

REMARKS: Values for the twenty-fourth root of two and its powers from equal-temperament tables computed by John H. Chalmers, Jr. The table is carried to nine octaves and a major third because this is the range of fundamental frequencies in a large organ. The numbering of the 24 tones starting with C, the customary reference-point, as 0 or 24, conforms to Julián Carrillo's numbering in Sistema General de Escritura Musical, and also to the numbering-practice of other authorities, even in some other tuning-systems besides quartertones. The signs in the Couper modification of the Vyshnegradski notation-system may be called: semisharp ♯, sharp #, sesquisharp 𝄪 ; semiflat 𝄳, flat ♭, sesquiflat 𝄫 . No prejudice against other quartertone nomenclatures is implied nor intended.

IVOR DARREG.

(for Pitch a'=440 Hz, c' = 261.626 Hz)

TWENTY-FOUR-TONE (QUARTERTONE) EQUAL TEMPERAMENT WHICH INCLUDES THE CUSTOMARY TWELVE-TONE EQUAL TEMPERAMENT: BEATS PER SECOND FOR CERTAIN INTERVALS.

FIFTHS (flat)			FOURTHS (sharp)			MAJOR THIRDS (sharp)			MINOR SIXTHS (flat)		
A	e	0.37	A	d	0.50	F	A	3.47	A	f	6.93
A𝄲	e𝄲	.38	A𝄲	d𝄲	.51	F𝄲	A𝄲	3.57	A𝄲	f𝄲	7.13
B♭	f	.39	B♭	e♭	.53	F♯	A♯	3.67	A♯	f♯	7.34
B𝄳	f𝄲	.41	B𝄳	e𝄳	.54	G𝄳	B𝄳	3.78	B𝄳	g𝄳	7.56
B♭	f♯	.42	B	e	.56	G	B	3.89	B	g	7.78
B𝄲	f♯♯	.43	B𝄲	e𝄲	.57	G𝄲	B𝄲	4.00	B𝄲	g𝄲	8.01
c	g	.44	c	f	.59	A♭	c	4.12	c	a♭	8.24
c𝄲	g𝄲	.46	c𝄲	f𝄲	.61	A𝄳	c𝄲	4.24	c𝄲	a𝄳	8.48
c♯	g♯	.47	c♯	f♯	.63	A	c♯	4.37	c♯	a	8.73
c♯	g♯	.48	c♯	f♯	.64	A𝄲	c♯♯	4.50	d𝄳	b𝄫	8.99
d	a	.50	d	g	.66	B♭	d	4.62	d	b♭	9.25
d𝄲	a𝄲	.51	d𝄲	g𝄲	.68	B𝄳	d𝄲	4.76	d𝄲	b𝄳	9.52
d♯	a♯	.53	d♯	g♯	.70	B	d♯	4.90	d♯	b	9.80
d♯	a♯	.54	d♯	g♯	.72	B𝄲	d♯♯	5.05	e𝄳	c′𝄳	10.09
e	b	.56	e	a	.74	c	e	5.19	e	c′	10.38
e𝄲	b𝄲	.57	e𝄲	a𝄲	.77	c𝄲	e𝄲	5.34	e𝄲	c′𝄲	10.68
f	c′	.59	f	b♭	.79	d♭	f	5.50	f	d′♭	11.00
f𝄲	c′𝄲	.61	f𝄲	b𝄳	.81	d𝄳	f𝄲	5.66	f𝄲	d′𝄳	11.32
f♯	c′♯	.63	f♯	b	.84	d	f♯	5.83	f♯	d′	11.65
f♯	c′♯	.64	f♯	b𝄲	.86	e𝄫	g𝄳	6.00	g𝄳	e′𝄫	11.99
g	d′	.66	g	c′	.89	e♭	g	6.17	g	e′♭	12.35
g𝄲	d′𝄲	.68	g𝄲	c′𝄲	.91	e𝄳	g𝄲	6.35	g𝄲	e′𝄳	12.71
g♯	d′♯	.70	g♯	c′♯	.94	e	g♯	6.54	g♯	e′	13.08
g♯	d′♯	.72	g♯	c′♯	.96	e𝄲	g♯♯	6.73	g♯	e′𝄲	13.46
a	e′	.74	a	d′	.99	f	a	6.93	a	f′	13.86
a𝄲	e′𝄲	.77	a𝄲	d′𝄲	1.02	f𝄲	a𝄲	7.13			
b♭	f′	.79	b♭	e′♭	1.05	f♯	a♯	7.34			
b𝄳	f′𝄲	.81	b𝄳	e′𝄳	1.08	g𝄳	b𝄳	7.56			
b	f′♯	.84	b	e′	1.12	g	b	7.78			
b𝄲	f′♯	.86	b𝄲	e′𝄲	1.15	g𝄲	b𝄲	8.01			
c′	g′	.89	c′	f′	1.18	a♭	c′	8.24			
c′𝄲	g′𝄲	.91	c′𝄲	f′𝄲	1.21	a𝄳	c′𝄲	8.48			
c′♯	g′♯	.94	c′♯	f′♯	1.25	a	c♯	8.73			
c′♯	g′♯	.96	c′♯	f′♯	1.29	a𝄲	c♯♯	8.99			
d′	a′	.99	d′	g′	1.33	b♭	d′	9.25			
d′𝄲	a′𝄲	1.02	d′𝄲	g′𝄲	1.37	b𝄳	d′𝄲	9.52			
d′♯	a′♯	1.05	d′♯	g♯	1.41	b	d′♯	9.80			
d′♯	a′♯	1.08	d′♯	g′♯	1.45	c′𝄳	e′𝄳	10.09			
e′	b′	1.12	e′	a′	1.48	c′	e′	10.38			
e′𝄲	b′𝄲	1.15	e′𝄲	a′𝄲	1.53	c′𝄲	e′𝄲	10.68			
f′	c″	1.18	f′	b′♭	1.58	d′♭	f′	11.00			
f′𝄲	c″𝄲	1.22	f′𝄲	b′𝄳	1.63	d′𝄳	f′𝄲	11.32			
f′♯	c″♯	1.25	f′♯	b′	1.67	d′	f′♯	11.65			
f′♯	c″♯	1.29	f′♯	b′𝄲	1.72	e′𝄫	g′𝄳	11.99			
g′	d″	1.33	g′	c″	1.77	e′♭	g′	12.35			
g′𝄲	d″𝄲	1.37	g′𝄲	c″𝄲	1.82	e′𝄳	g′𝄲	12.71			
g′♯	d″♯	1.41	g′♯	c″♯	1.88	e′	g′♯	13.08			
g′♯	d″♯	1.45	g′♯	c″♯	1.93	e′𝄲	g′♯	13.46			
a′	e″	1.48	a′	d″	1.98	f′	a′	13.86			

(for Pitch a' = 440 Hz, c' = 261.626 Hz)

TWENTY-FOUR-TONE (QUARTERTONE) EQUAL TEMPERAMENT
BEATS PER SECOND FOR CERTAIN INTERVALS, CONTINUED

MINOR THIRDS (flat)			MAJOR SIXTHS (sharp)			SUBMINOR SEVENTHS (flat)			SEMIAUGMENTED FOURTHS (flat)		
F	A𝄭	4.71	F	d	3.96	C	B𝄫	4.95	c	f𝄲	1.10
F𝄲	A𝄳	4.85	F𝄲	d𝄲	4.08	C𝄲	B𝄭	5.10	c𝄲	f#	1.12
F#	A	4.99	F#	d#	4.20	C#	B𝄳	5.25	c#	f##	1.16
F##	A𝄲	5.14	G𝄳	e𝄳	4.32	C##	B	5.40	d𝄳	g	1.19
G	B𝄭	5.29	G	e	4.45	D	c𝄳	5.56	d	g𝄲	1.23
G𝄲	B𝄳	5.45	G𝄲	e𝄲	4.58	D𝄲	c	5.72	d𝄲	g#	1.27
G#	B	5.60	A𝄭	f	4.71	D#	c𝄲	5.89	d#	g##	1.30
G##	B𝄲	5.76	A𝄳	f𝄲	4.85	D##	c#	6.06	e𝄳	a	1.34
A	c	5.94	A	f#	4.99	E	d𝄳	6.24	e	a𝄲	1.38
A𝄲	c𝄲	6.11	A𝄲	f##	5.14	E𝄲	d	6.42	e𝄲	a#	1.42
A#	c#	6.29	B𝄭	g	5.29	F	e𝄫	6.61	f	b𝄳	1.46
A##	c##	6.47	B𝄳	g𝄲	5.45	F𝄲	e𝄭	6.80	f𝄲	b	1.50
B	d	6.66	B	g#	5.60	F#	e𝄳	7.00	f#	b𝄲	1.55
B𝄲	d𝄲	6.86	B𝄲	g##	5.76	F##	e	7.21	g𝄳	c′	1.59
c	e𝄭	7.06	c	a	5.94	G	f𝄳	7.42	g	c′𝄲	1.64
c𝄲	e𝄳	7.27	c𝄲	a𝄲	6.11	G𝄲	f	7.64	g	c′#	1.69
c#	e	7.48	c#	a#	6.29	G#	f𝄲	7.86	g#	c′##	1.74
c##	e𝄲	7.70	d𝄳	b𝄳	6.47	G##	f#	8.09	a𝄳	d′	1.79
d	f	7.92	d	b	6.66	A	g𝄳	8.33	a	d′𝄲	1.84
d𝄲	f𝄲	8.15	d𝄲	b𝄲	6.86	A𝄲	g	8.57	a𝄲	d′#	1.89
d#	f#	8.39	e𝄭	c′	7.06	A#	g𝄲	8.82	a#	d′##	1.95
e𝄳	g𝄳	8.64	e𝄳	c′𝄲	7.27	A##	g#	9.08	b𝄳	e′	2.01
e	g	8.89	e	c′#	7.48	B	a𝄳	9.35	b	e′𝄲	2.07
e𝄲	g𝄲	9.15	e	c′##	7.70	B𝄲	a	9.62	c′𝄳	f′	2.13
f	a𝄭	9.42	f	d′	7.92	c	b𝄫	9.90	c′	f𝄲	2.19
f𝄲	a𝄳	9.70	f𝄲	d′𝄲	8.15						
f#	a	9.98	f#	d′#	8.39						
g𝄳	b𝄫	10.27	g𝄳	e′𝄳	8.64						
g	b𝄭	10.58	g	e′	8.89						
g𝄲	b𝄳	10.89	g𝄲	e′𝄲	9.15						
g#	b	11.21	a𝄭	f′	9.42						
g##	b𝄲	11.52	a𝄳	f′𝄲	9.70						
a	c′	11.87	a	f′#	9.98						
a𝄲	c′𝄲	12.22	a𝄲	f′##	10.27						
a#	c′#	12.58	b𝄫	g′	10.58						
b𝄳	d′𝄳	12.95	b𝄳	g′𝄲	10.89						
b	d′	13.33	b	g′#	11.21						
b𝄲	d′𝄲	13.72	b𝄲	g′##	11.52						
c′	e′𝄭	14.12	c′	a′	11.87						
c′𝄲	e′𝄳	14.53	c′𝄲	a′𝄲	12.22						
c′#	e′	14.96	c′#	a′#	12.58						
			c′##	a′##	12.95						
			d′	b′	13.33						
			e′𝄫	c″𝄳	13.72						
			e′𝄭	c″	14.12						

The quartertone notation used in these tables is the Couper modification of Vyshnegradski's notation, and its use here is not intended to imply any prejudicial attitude toward the many other quartertone notations in existence.

The term "subminor seventh" is intended to mean the interval of 19 quartertones' width or 950 cents, which develops beats because it approximates the just subminor seventh, ratio 4:7, of 969 cents' width.

The term "semiaugmented fourth" is intended to mean the interval 11 quartertones in width, 550 cents, imitating the just interval of ratio 8:11, width 551 cents.

Twelve-tone system beats have been checked against published tables.