

## Welcome to my collection of n-uniform tilings!

Prior to my research in the area of n-uniform tilings, no complete sets of n-uniform tilings had ever been enumerated for any  $n > 3$ . I have long been fascinated by these kinds of tilings, and after a great deal of effort, I was able to write a computer program that automatically generates complete sets of tilings for a given  $n$ . It took approximately one month of computation time to generate the 6-uniform tilings. The program is also able to generate, in considerably less time, sets of m-Archimedean, n-uniform tilings, where  $m$  is close to  $n$ . This capability was used to reproduce the results of Otto Krottenheerdt, and to generate his tilings (where  $m=n$ ).

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An n-uniform tiling is an edge-to-edge tiling of regular polygons having  $n$  distinct transitivity classes of vertices. A uniform tiling is 1-uniform, where all vertices belong to the same transitivity class. This means that each vertex of the tiling is equivalent to every other vertex with respect to the symmetries (translation, rotation, flip) of the tiling. So for example, if I picked one vertex, and you picked another, we would be able to shift, rotate, and/or flip a copy of the tiling in such a way as to not only align our two vertices, but also to perfectly map the copy onto the original tiling. (Imagine two transparencies where all the lines of the tilings coincide exactly.) A 2-uniform tiling will have vertices that we could label A, and others that we could label B. Each A vertex can be mapped onto every other A vertex, but cannot be mapped to any B vertex. (We could shift, rotate, and/or flip a B onto an A, but the tiling and the copy will never align.)

Sloane's A068599: Gives Number of n-Uniform Tilings: 11, 20, 61, 151, 332, 673, ...

- The Uniform Tilings (11)
- The 2-Uniform Tilings (20)
- The 3-Uniform Tilings (61)
- The 4-Uniform Tilings (151)
- The 5-Uniform Tilings (332)
- The 6-Uniform Tilings (673)

An n-Archimedean tiling is a tiling having  $n$  distinct vertex types, with "vertex type" referring to the type and order of polygons surrounding each vertex in the tiling. A standard notation used to denote a specific vertex type gives the number of sides of each polygon separated by periods. So for example, a vertex surrounded by a square, a hexagon, and a dodecagon (either clockwise or counterclockwise) would be given as "4.6.12". The vertex type 3.3.6.6 is considered distinct from the vertex type 3.6.3.6 because the order of surrounding polygons is different. One could not map 3.3.6.6 onto 3.6.3.6 by any rotation or flip.

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Sloane's A068600: Gives Number of n-Archimedean, n-Uniform Tilings (Krottenheerdt Tilings):

11, 20, 39, 33, 15, 10, 7, 0, 0, 0, ...

- The 3-Archimedean, 3-Uniform Tilings (39)
- The 4-Archimedean, 4-Uniform Tilings (33)
- The 5-Archimedean, 5-Uniform Tilings (15)
- The 6-Archimedean, 6-Uniform Tilings (10)
- The 7-Archimedean, 7-Uniform Tilings (7)

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### Numbers of Tilings

n	m-Archimedean
1	11
2	20
3	39
4	33
5	15
6	10
7	7
8	0
9	0
10	0

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	>14	Total
1	11	0	0	0	0	0	0	0	0	0	0	0	0	0	0	11
2	0	20	0	0	0	0	0	0	0	0	0	0	0	0	0	20
3	0	22	39	0	0	0	0	0	0	0	0	0	0	0	0	61
4	0	33	85	33	0	0	0	0	0	0	0	0	0	0	0	151
5	0	74	149	94	15	0	0	0	0	0	0	0	0	0	0	332
6	0	100	284	187	92	10	0	0	0	0	0	0	0	0	0	673
7	0	?	?	?	?	?	7	0	0	0	0	0	0	0	0	?
8	0	?	?	?	?	?	20	0	0	0	0	0	0	0	0	?
9	0	?	?	?	?	?	?	8	0	0	0	0	0	0	0	?
10	0	?	?	?	?	?	?	27	0	0	0	0	0	0	0	?
11	0	?	?	?	?	?	?	?	1	0	0	0	0	0	0	?
12	0	?	?	?	?	?	?	?	?	0	0	0	0	0	0	?
13	0	?	?	?	?	?	?	?	?	?	?	?	0	0	0	?
14	0	?	?	?	?	?	?	?	?	?	?	?	?	0	0	?
>14	0	?	?	?	?	?	?	?	?	?	?	?	?	?	0	?
Total	11	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	0	∞

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