Notes on Terminology (continued)

5. $\binom{3}{6}$ or $(3,6)$ = 3 out of 6

6. (4,8) hebdomekontany = a combination-product set of 70 members, derived by taking combinations of 4-out-of-8 elements (usually the harmonics 1 3 5 7 9 11 13 15, but they could be any appropriate set of 8) and multiplying together the 4 elements of each combination, thus; 1·3·5·7, 1·3·5·9, 1·3·5·11, etc., to 9·11·13·15.

Rough Draft

Partitioned Cross-Sets of the Hebdomekontany

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Item 1. In the 4)8 Hebdomekontany the 1)2 dyany subset has 28 varieties, each of which occurs 20 times, in a 3)6 Eikosany partitioned cross-set.

Example; the $\binom{1}{2}$ 1 3 dyany × the $\binom{3}{6}$ 5 7 9 11 13 15 Eikosany,

$$\left\{\binom{1}{2} 1\ 3\right\} \times \left\{\binom{3}{6} 5\ 7\ 9\ 11\ 13\ 15\right\} =$$

×	1	3
5·7·9	1·5·7·9	3·5·7·9
5·7·11	1·5·7·11	3·5·7·11
5·7·13	1·5·7·13	3·5·7·13
5·7·15	1·5·7·15	3·5·7·15
5·9·11	1·5·9·11	3·5·9·11
5·9·13	1·5·9·13	3·5·9·13
5·9·15	1·5·9·15	3·5·9·15
5·11·13	1·5·11·13	3·5·11·13
5·11·15	1·5·11·15	3·5·11·15
5·13·15	1·5·13·15	3·5·13·15
7·9·11	1·7·9·11	3·7·9·11
7·9·13	1·7·9·13	3·7·9·13
7·9·15	1·7·9·15	3·7·9·15
7·11·13	1·7·11·13	3·7·11·13
7·11·15	1·7·11·15	3·7·11·15
7·13·15	1·7·13·15	3·7·13·15
9·11·13	1·9·11·13	3·9·11·13
9·11·15	1·9·11·15	3·9·11·15
9·13·15	1·9·13·15	3·9·13·15
11·13·15	1·11·13·15	3·11·13·15

Part II is added as an introduction, So read that first.

This material is developed from letters to Adriaan Fokker 1970 and John Chalmers 1971 (on file). Also used as sources; New Math, by Cole Swensen, Quill (from that period but re-issued 1988 in paperback) and Finite Mathematics by Seymour Lipschutz, Schaum's Outline Series, McGraw-Hill, 1966.

Item 2. The $\left\{\binom{4}{8}\ 1\ 3\ 5\ 7\ 9\ 11\ 13\ 15\right\}$ Hebdomekontany may be partitioned into the following 13 illustrative types of cross-sets;

	variations or permutations of Partition
$\left\{\binom{0}{0}\ \emptyset\right\}(1y) \times \left\{\binom{4}{8}\ a\,b\,c\,d\,e\,f\,g\,h\right\}(70y)$	1
$\left\{\binom{0}{1}\ a\right\}(1y) \times \left\{\binom{4}{7}\ b\,c\,d\,e\,f\,g\,h\right\}(35y)$	8
$\left\{\binom{1}{1}\ a\right\}(1y) \times \left\{\binom{3}{7}\ b\,c\,d\,e\,f\,g\,h\right\}(35y)$	8
$\left\{\binom{0}{2}\ a\,b\right\}(1y) \times \left\{\binom{4}{6}\ c\,d\,e\,f\,g\,h\right\}(15y)$	28
$\left\{\binom{1}{2}\ a\,b\right\}(2y) \times \left\{\binom{3}{6}\ c\,d\,e\,f\,g\,h\right\}(20y)$	28
$\left\{\binom{2}{2}\ a\,b\right\}(1y) \times \left\{\binom{2}{6}\ c\,d\,e\,f\,g\,h\right\}(15y)$	28
$\left\{\binom{0}{3}\ a\,b\,c\right\}(1y) \times \left\{\binom{4}{5}\ d\,e\,f\,g\,h\right\}(5y)$	56
$\left\{\binom{1}{3}\ a\,b\,c\right\}(3y) \times \left\{\binom{3}{5}\ d\,e\,f\,g\,h\right\}(10ny)$	56
$\left\{\binom{2}{3}\ a\,b\,c\right\}_{3ny} \times \left\{\binom{2}{5}\ d\,e\,f\,g\,h\right\}_{10ny}$	56
$\left\{\binom{3}{3}\ a\,b\,c\right\}_{1ny} \times \left\{\binom{1}{5}\ d\,e\,f\,g\,h\right\}_{5ny}$	56
$\left\{\binom{0}{4}\ a\,b\,c\,d\right\}_{1ny} \times \left\{\binom{4}{4}\ e\,f\,g\,h\right\}_{1ny}$	70
$\left\{\binom{1}{4}\ a\,b\,c\,d\right\}_{4ny} \times \left\{\binom{3}{4}\ e\,f\,g\,h\right\}_{4ny}$	70
$\left\{\binom{2}{4}\ a\,b\,c\,d\right\}_{6ny} \times \left\{\binom{2}{4}\ e\,f\,g\,h\right\}_{6ny}$	$\frac{70}{(2)} = (35)$

12 remaining cross-sets (total 25) are in effect a re-ordering of the first 12 cross-sets above.

5
(1, 5)
|
(3, 3)

Item 3; In the 4-out-of-8, (4,8) hebdomekontany matrix the (1,5) Pentany subset has 56 variations, each of which occurs once, in a (3,3) Monany, partitioned complementary cross-set. Example; the $\binom{1}{5}$ 1 3 5 7 9 Pentany partition/cross with $\binom{3}{3}$ 11 13 15 Monany can be expressed,

$$\left\{\binom{1}{5} 1\,3\,5\,7\,9\right\} \times \left\{\binom{3}{3} 11\ 13\ 15\right\} =$$

×	1	3	5	7	9
11·13·15	1·11·13·15	3·11·13·15	5·11·13·15	7·11·13·15	9·11·13·15

5
(4, 5)
|
(0, 3)

Item 4; In the (4,8) Hebdomekontany matrix the (4, 5) Pentany subset has 56 variations, each of which occurs once, in a cross-set with its partitioned complementary (0, 3) monany. Note; because the (4, 5) Pentany conceals a subharmonic hexad, it is helpful to show that additional relationship, thus — Example; $\left\{\binom{4}{5} 1\,3\,5\,7\,9\right\} \times \left\{\binom{0}{3} 11\ 13\ 15\right]$,

added step: $\overline{1}$ $\overline{3}$ $\overline{5}$ $\overline{7}$ $\overline{9}$ (subharmonic Pentad)

× 1·3·5·7·9 =

×	3·5·7·9	1·5·7·9	1·3·7·9	1·3·5·9	1·3·5·7	$\binom{4}{5}$set
(empty) ∅	3·5·7·9	1·5·7·9	1·3·7·9	1·3·5·9	1·3·5·7	

The last step of this operation, (multiplication of of the (4 5) Pentany by the (0 3) Monany [an empty set, ~~indicated ∅~~]) is obviously academic.

4 (1, 4) 4 (3, 4)

Item 5; In the 4-out-of-8, (4, 8) Hebdomekontany matrix the (1, 4) Tetrany forms a partitioned cross-set with the (3, 4) Tetrany. There are 70 combinations by which this partitioning may occur. Note; the (3, 4) Tetrany carries a concealed sub-harmonic tetrad which is identified thus; <u>Example</u>

$$(\overline{1} \quad \overline{3} \quad \overline{5} \quad \overline{7}) \times 1\cdot3\cdot5\cdot7 =$$

×	3·5·7	1·5·7	1·3·7	1·3·5 ← (3,4) Tetrany
(1,4) Tetrany — 9	3·5·7·9	1·5·7·9	1·3·7·9	1·3·5·9
11	3·5·7·11	1·5·7·11	1·3·7·11	1·3·5·11
13	3·5·7·13	1·5·7·13	1·3·7·13	1·3·5·13
15	3·5·7·15	1·5·7·15	1·3·7·15	1·3·5·15

Now, then — By adding a single tone, 1·3·5·7, to the 4 harmonic tetrads and to the 4 subharmonic tetrads, a set of 4 plus 4 Pentads will occur, each of which shares the 1·3·5·7 in common. A 17-tone quasi-diamond, providing 8 ways to harmonize the member 1·3·5·7. If the entire set above is divided by 1·3·5·7 this simple relationship emerges;

$\overline{9}$, $\overline{11}$, $\overline{13}$, $\overline{15}$ add

× $\overline{1\cdot3\cdot5\cdot7}$ =	$\overline{1}$	$\overline{3}$	$\overline{5}$	$\overline{7}$	
9	9/1	9/3	9/5	9/7	9/9
11	11/1	11/3	11/5	11/7	11/11
13	13/1	13/3	13/5	13/7	13/13
15	15/1	15/3	15/5	15/7	15/15
add → 1, 3, 5, 7	1/1	3/3	5/5	7/7	

One sees (by $\frac{1\cdot3\cdot5\cdot7}{1\cdot3\cdot5\cdot7} = 1/1$) the 8 harmonizing roles that 1·3·5·7 can play; 4 harmonic, partitioned w. 4 subharmonic senses.

ITem 5 continued ;

Another way to show this is to simply add 1·3·5·7 to each of the tetrads, thus

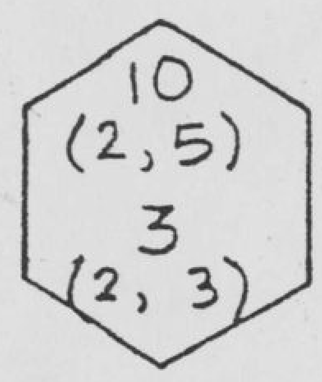
	($\overline{1}$)	($\overline{3}$)	($\overline{5}$)	($\overline{7}$)	$\overline{9}$ $\overline{11}$ $\overline{13}$ $\overline{15}$ sequence
	3·5·7	1·5·7	1·3·7	1·3·5	add ↓
9	3·5·7·9	1·5·7·9	1·3·7·9	1·3·5·9	1·3·5·7
11	3·5·7·11	1·5·7·11	1·3·7·11	1·3·5·11	1·3·5·7
13	3·5·7·13	1·5·7·13	1·3·7·13	1·3·5·13	1·3·5·7
sequence 15	3·5·7·15	1·5·7·15	1·3·7·15	1·3·5·15	1·3·5·7
1 3 5 7 add	1·3·5·7	1·3·5·7	1·3·5·7	1·3·5·7	

The horizontal lines may be seen as partitioned cross-sets {(4, 5) a b c d e} × {(0, 3) f g h} .

The vertical lines may be seen as partitioned cross-sets {(1, 5) ab c de} × {(3, 3) f g h} .

But that doesn't really tie it all together as well as I'd wish. —

What we have in this "quasi-diamond" is that set of (4, 5) Pentanies horizontal / which share 1·3·5·7. This simultaneously gives us the material for that set of (1, 5) Pentanies vertical / which, also share the 1·3·5·7. These are the 17 points that would be shared by the ogdoadic cross-set (1 3 5 7 9 11 13 15) × ($\overline{1}$ $\overline{3}$ $\overline{5}$ $\overline{7}$ $\overline{9}$ $\overline{11}$ $\overline{13}$ $\overline{15}$) with its 1/1 at key 1·3·5·7. [This kind of cross-set is usually called a "Diamond" after Partch's usage.] These 17 points (in their 70 partitioned permutations) are the 70 bonding-sites between centered and centerless modules in ogdoadic tone-space. They are the basis for modulating from Hebdomekontanies to 8adic Diamonds & visa/versa thru-out the infinite realms of open 8adic tone-space.

Item 6; 

within the (4,8) hebdomekontany the (2,5) Dekany forms a partitioned cross-set with the (2,3) Triany. There are 56 ways the 8 elements of the master set can be partitioned into 2 sets of 3 and 5 elements.

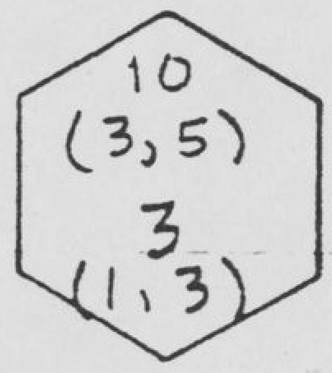
Example;

$$\{(2,5)\ 1\ 3\ 5\ 7\ 9\} \times \{(2,3)\ 11\ 13\ 15\} =$$

(2,5) Dekany ↓ ; $\xrightarrow{\quad}$ (2,3) triany

×	($\overline{15}$) 11·13	($\overline{13}$) 11·15	($\overline{11}$) 13·15
1·3	1·3·11·13	1·3·11·15	1·3·13·15
1·5	1·5·11·13	1·5·11·15	1·5·13·15
1·7	1·7·11·13	1·7·11·15	1·7·13·15
1·9	1·9·11·13	1·9·11·15	1·9·13·15
3·5	3·5·11·13	3·5·11·15	3·5·13·15
3·7	3·7·11·13	37·11·15	3·7·13·15
3·9	3·9·11·13	3·9·11·15	3·9·13·15
5·7	5·7·11·13	5·7·11·15	5·7·13·15
5·9	5·9·11·13	5·9·11·15	5·9·13·15
7·9	7·9·11·13	7·9·11·15	7·9·13·15

Item 7;

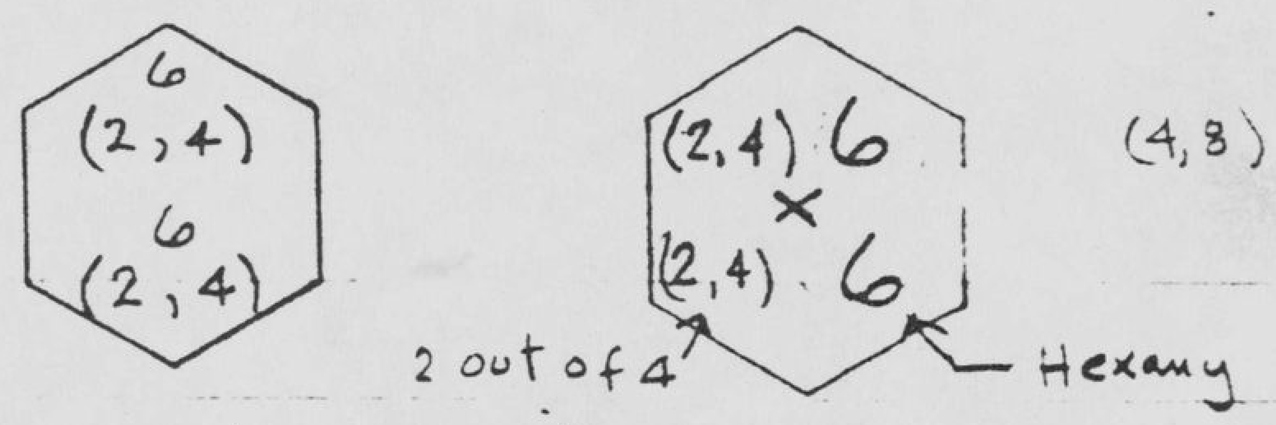


In the (4,8) Hebdomekontany [70ny] matrix the (3,5) Dekany [10ny] forms a partitioned cross-set with the (1,3) Triany [3ny]. The are 56 ways a set of 8 elements may partitioned into 2 sets, of 5 and 3 elements ~~each~~. (Hence 56 cross-sets)

Example; $\{(3,5)\ 1\ 3\ 5\ 7\ 9\} \times \{(1,3)\ 11\ 13\ 15\}$

×	11	13	15
1·3·5	1·3·5·11	1·3·5·13	1·3·5·15
1·3·7	1·3·7·11	1·3·7·13	1·3·7·15
1·3·9	1·3·9·11	1·3·9·13	1·3·9·15
1·5·7	1·5·7·11	1·5·7·13	1·5·7·15
1·5·9	1·5·9·11	1·5·9·13	1·5·9·15
1·7·9	1·7·9·11	1·7·9·13	1·7·9·15
3·5·7	3·5·7·11	3·5·7·13	3·5·7·15
3·5·9	3·5·9·11	3·5·9·13	3·5·9·15
3·7·9	3·7·9·11	3·7·9·13	3·7·9·15
5·7·9	5·7·9·11	5·7·9·13	5·7·9·15

Item 8:



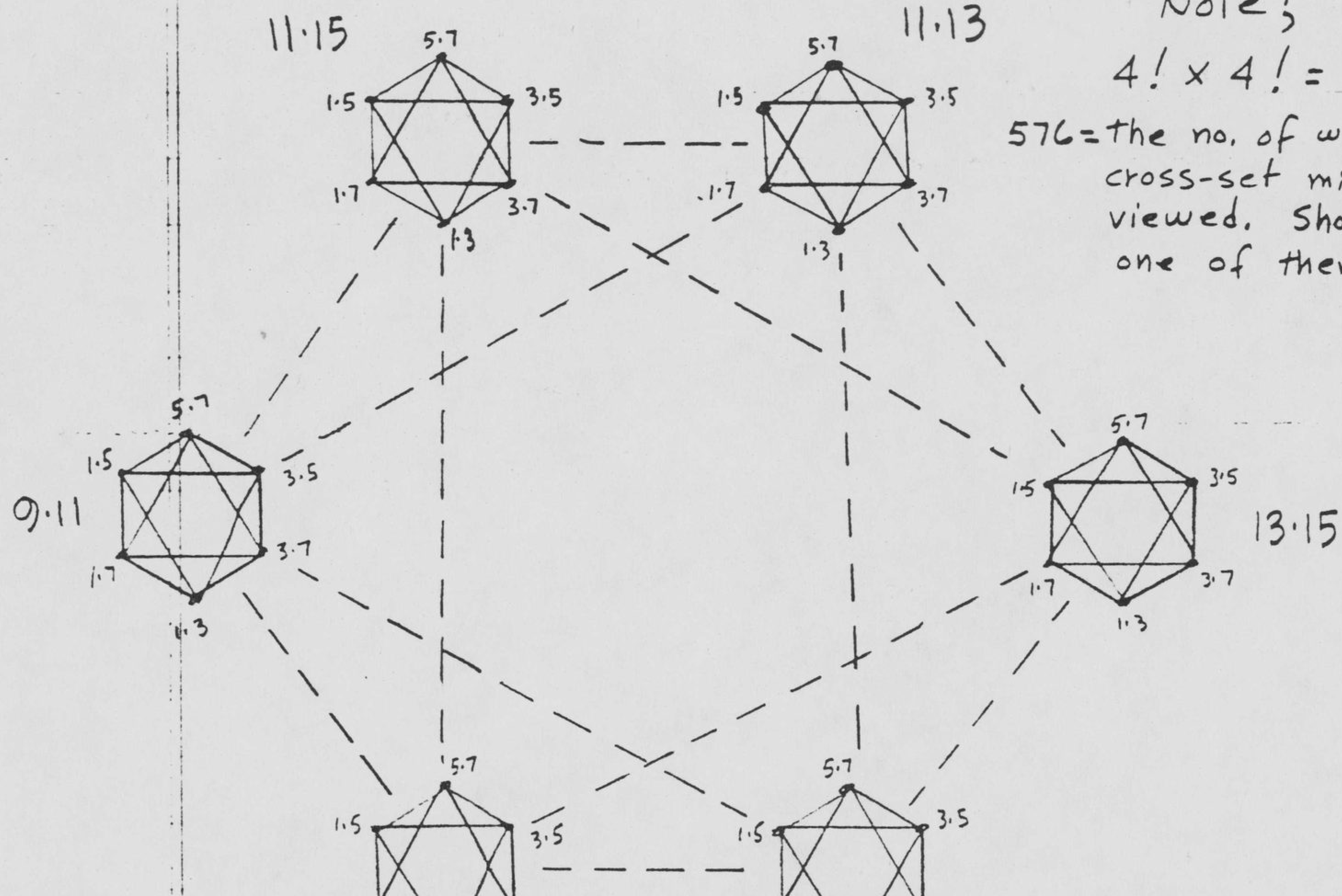
(4,8) 70

In the (4,8) Hebdomekontany matrix the (2,4) Hexany forms a partitioned cross-set with the (2,4) Hexany. The master-set of 8 elements may be partitioned into 2 sets, of 4 and 4 elements ~~each~~, in 35 ways only (70 ÷ 2 = 35), because of the mirroring nature of the entire theoretical group of 70 ways.

Example; {(2,4) 1 3 5 7} × {(2,4) 9 11 13 15}

×	1·3	1·5	1·7	3·5	3·7	5·7
9·11	1·3·9·11	1·5·9·11	1·7·9·11	3·5·9·11	3·7·9·11	5·7·9·11
9·13	1·3·9·13	1·5·9·13	1·7·9·13	3·5·9·13	3·7·9·13	5·7·9·13
9·15	1·3·9·15	1·5·9·15	1·7·9·15	3·5·9·15	3·7·9·15	5·7·9·15
11·13	1·3·11·13	1·5·11·13	1·7·11·13	3·5·11·13	3·7·11·13	5·7·11·13
11·15	1·3·11·15	1·5·11·15	1·7·11·15	3·5·11·15	3·7·11·15	5·7·11·15
13·15	1·3·13·15	1·5·13·15	1·7·13·15	3·5·13·15	3·7·13·15	5·7·13·15

Note;
$4! \times 4! = 576$
576 = the no. of ways this cross-set might be viewed. Shown, is but one of them.



2-out-of-6 → (2,6) 15 ← Pentadekany
× ← crossed with
2-out-of-2 → (2,2) 1 ← Monany

Item 9;

Contained In the hebdomekontany matrix, the (2,6) Pentadekany forms a partitioned cross-set with the (2,2) Monany. The ogdoad (8ad) partitions into 2 sets, of 6 and 2 elements, in 28 different ways. (Hence 28 different forms of this cross-set.)

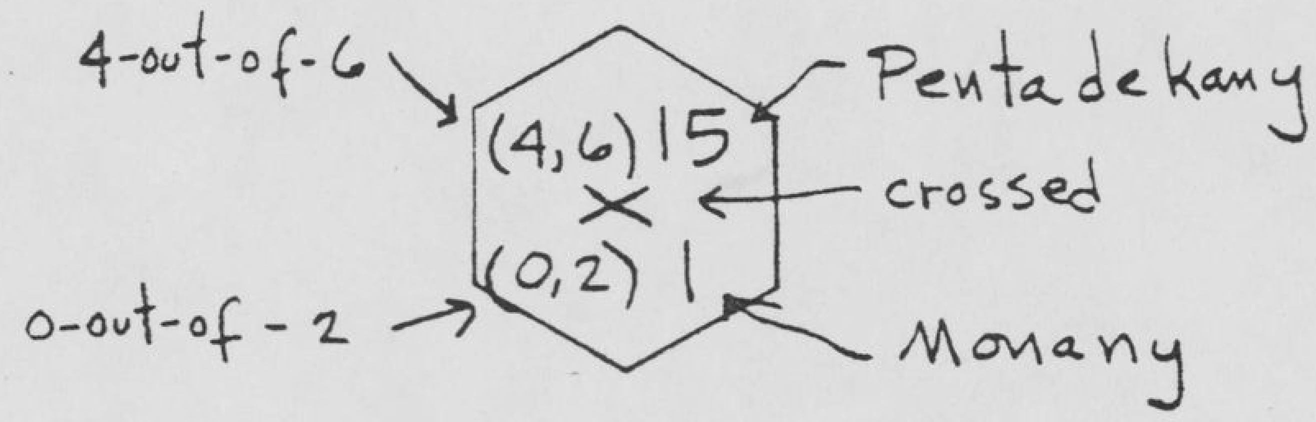
$\{(2,6)\ 1\ 2\ 5\ 7\ 9\ 11\} \times \{(2,2)\ 13\ 15\}$

Example;

×	13·15
1·3	1·3·13·15
1·5	1·5·13·15
1·7	1·7·13·15
1·9	1·9·13·15
1·11	1·11·13·15
3·5	3·5·13·15
3·7	3·7·13·15
3·9	3·9·13·15
3·11	3·11·13·15
5·7	5·7·13·15
5·9	5·9·13·15
5·11	5·11·13·15
7·9	7·9·13·15
7·11	7·11·13·15
9·11	9·11·13·15

receptor

Item 10;



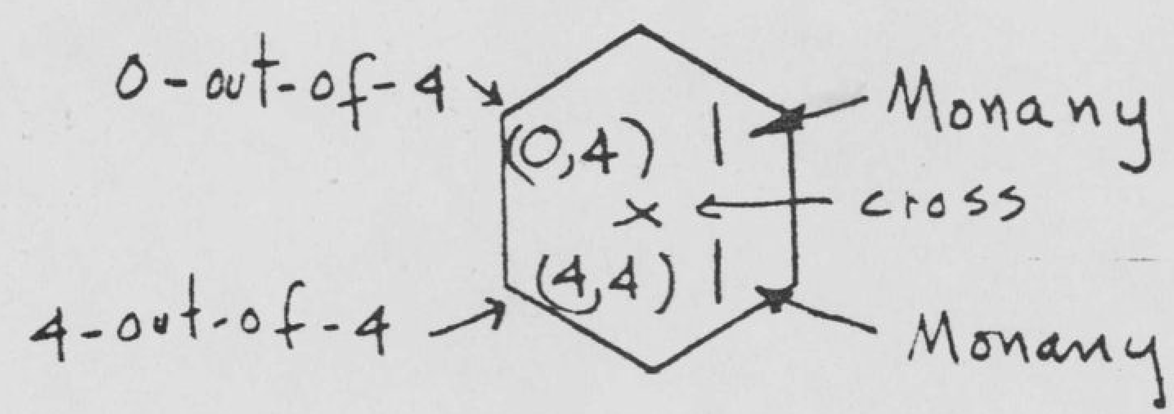
In the (4,8) Hebdomekontany [70ny] the (4,6) Pentadekany forms a partition cross-set with the (0,2) Monany. There are 28 expressions of this cross-set, based on the 28 ways 6 and 2 elements may be partitioned out of 8 elements.

Example; $\{(4,6)\ 1\ 3\ 5\ 7\ 9\ 11\} \times \{(0,2)\ 13\ 15\}$

×	∅ (= empty set)
1·3·5·7	1·3·5·7
1·3·5·9	1·3·5·9
1·3·5·11	1·3·5·11
1·3·7·9	1·3·7·9
1·3·7·11	1·3·7·11
1·3·9·11	1·3·9·11
1·5·7·9	1·5·7·9
1·5·7·11	1·5·7·11
1·5·9·11	1·5·9·11
1·7·9·11	1·7·9·11
3·5·7·9	3·5·7·9
3·5·7·11	3·5·7·11
3·5·9·11	3·5·9·11
3·7·9·11	3·7·9·11
5·7·9·11	5·7·9·11

Note; Caution – the "empty set" (∅) is not equivalent to zero (0). This notwithstanding, the cross is academic, and done to maintain a useful format.

Item 11;



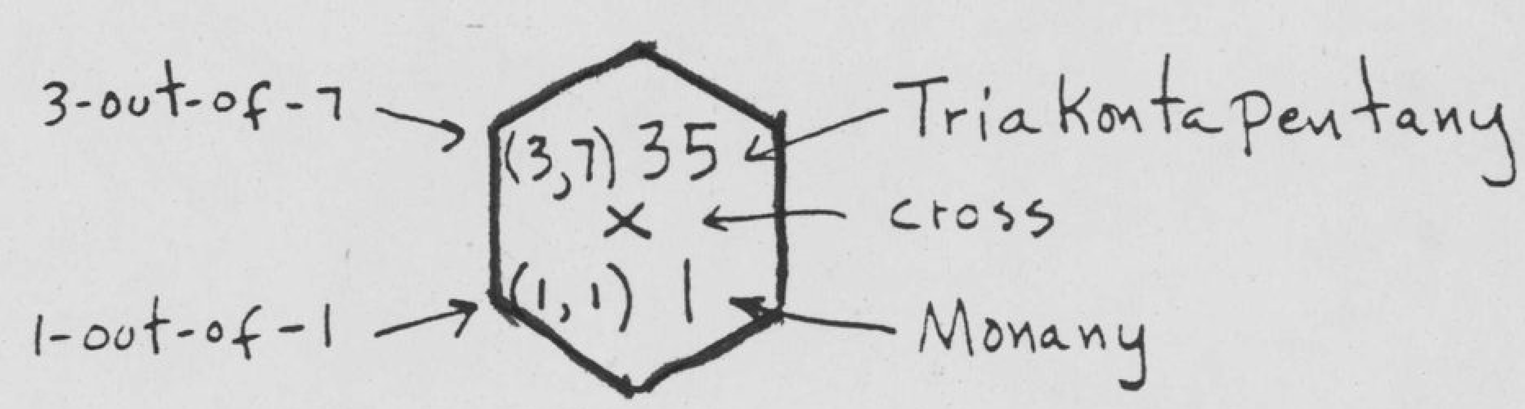
In the (4,8) Hebdomekontany [70ny] the (0,4) Monany forms a partitioned cross-set with the (4,4) Monany.
There are 70 expressions of this cross-set, according to the number of ways 4 and 4 elements may be partitioned from 8 elements.

Example; $\{(0,4)\ 1\ 3\ 5\ 7\} \times \{(4,4)\ 9\ 11\ 13\ 15\}$

×	9·11·13·15
∅	9·11·13·15

Note; obviously, going thru all 70 expressions of this cross-set would be a futile exercise. Dont do it.

ITem 12;



In the (4,8) Hebdomekonty the (3,7) TriaKontapentany forms a partitioned cross-set with the (1,1) Monany.
There are 8 expressions of this cross-set, according to the number of ways 7 and 1 elements may be partitioned from 8 elements.

Example; $\{(3,7)\ 1\ 3\ 5\ 7\ 9\ 11\ 13\} \times \{(1,1)\ 15\}$

×	15
1·3·5	1·3·5·15
1·3·7	1·3·7·15
1·3·9	1·3·9·15
etc	etc

Item 13;

(4,7) 35
×
(0,1) 1

In the (4,8) 70ny the (4,7) 35ny forms a partitioned cross-set with the (0,1) 1ny. There are 8 expressions of this cross-set, as there are 8 ways in which a set of 8 elements may be partitioned into 2 sets, of 7 and 1 elements.

Example; {(4,7) 1 3 5 7 9 11 13} × {(0,1) 15}

	Ø = (empty)
1·3·5·7	1·3·5·7
1·3·5·9	1·3·5·9
1·3·5·11	1·3·5·11
etc	etc

Item 14;

(4,8) 70
×
(0,0) 1

And finally, in the (4,8) 70ny, the (4,8) 70ny, itself, forms a partitioned cross-set with the (0,0) 1ny. There is only 1 expression of this cross-set. This corresponds to the 1 way a set of 8 elements may be partitioned into 2 sets, of 8 and 1 elements.

Example: {(4,8) 1 3 5 7 9 11 13 15} × {(0,0) Ø (= empty)}

	Ø (= empty)
1·3·5·7	1·3·5·7
1·3·5·9	1·3·5·9
1·3·5·11	1·3·5·11
etc	

Start Here

Part II
Partitioned Cross-sets of the Hexany

Pascal's Triangle;

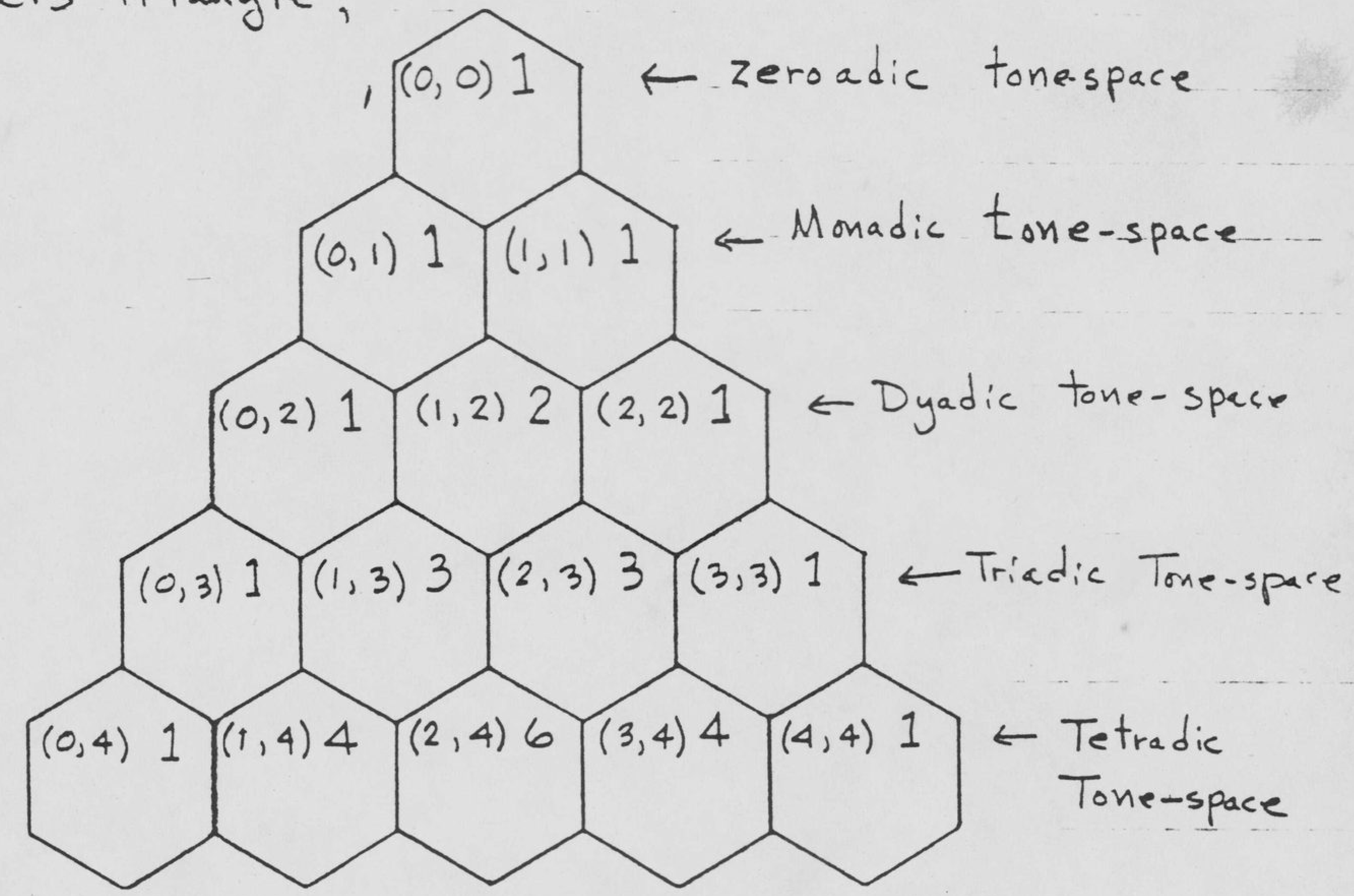
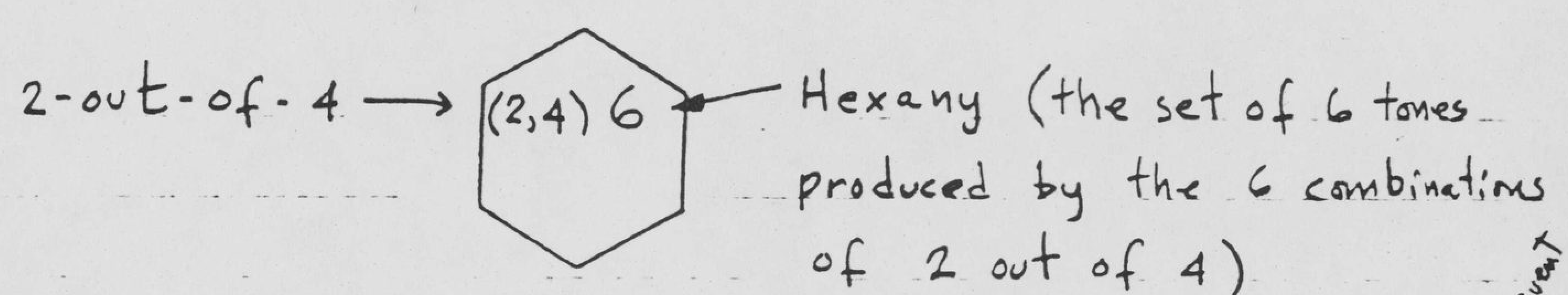


Fig 1.

Pascal's Triangle (Fig 1) may be used to codify the combination-product sets.



A. Each Base combination-product set contains all the subsequent combination-product sets obliquely above it in the triangular heirarchy.

B. Further, each "subsequent" combination-product set occurs a predictable number of times in the "base" combination-product set. The pattern of this multiple occurence is (interestingly enough) also a combination-product set. A cross-set is produced. The second set is labeled "consequent" set.

C. The partitioned nature of the cross-set is seen in the numerical terms of combination as well as in the elements of the set. To illustrate;

	"Terms" (1st, 2nd)	"Elements"	
"Subsequent set"	(1 out of 3)	1 3 5	The partitioned cross-sets
"Consequent set"	(1 out of 1)	7	
"Base set"	(2 out of 4)	1 3 5 7	

Partitioned into 1 & 1

Partitioned into 3 & 1

Partitioned into (1 3 5) & (7)

D. Finally, the partitioned cross-set, itself, has a predictable number of "expressions" (permutations) based on the no. of ways the elements of the "base set" can be partitioned into the appropriate no. of elements in the partitioned sets. For example; when the "base" set has the 4 elements (1 3 5 7), and is being partitioned into sets of 3 and 1 elements, the "expressions" or permutations are (135) (7), (1 3 7) (5), (1 5 7) (3), and (3 5 7) (1); 4 permutations.

E. When the various "consequent" sets of a "base" set are assigned to their proper niches in the Triangle they produce, among them, the 180° rotation of the Triangle (see Fig 2). The apex has now become the nadir, which is juxtaposed upon the "base" set, (see Fig 3).

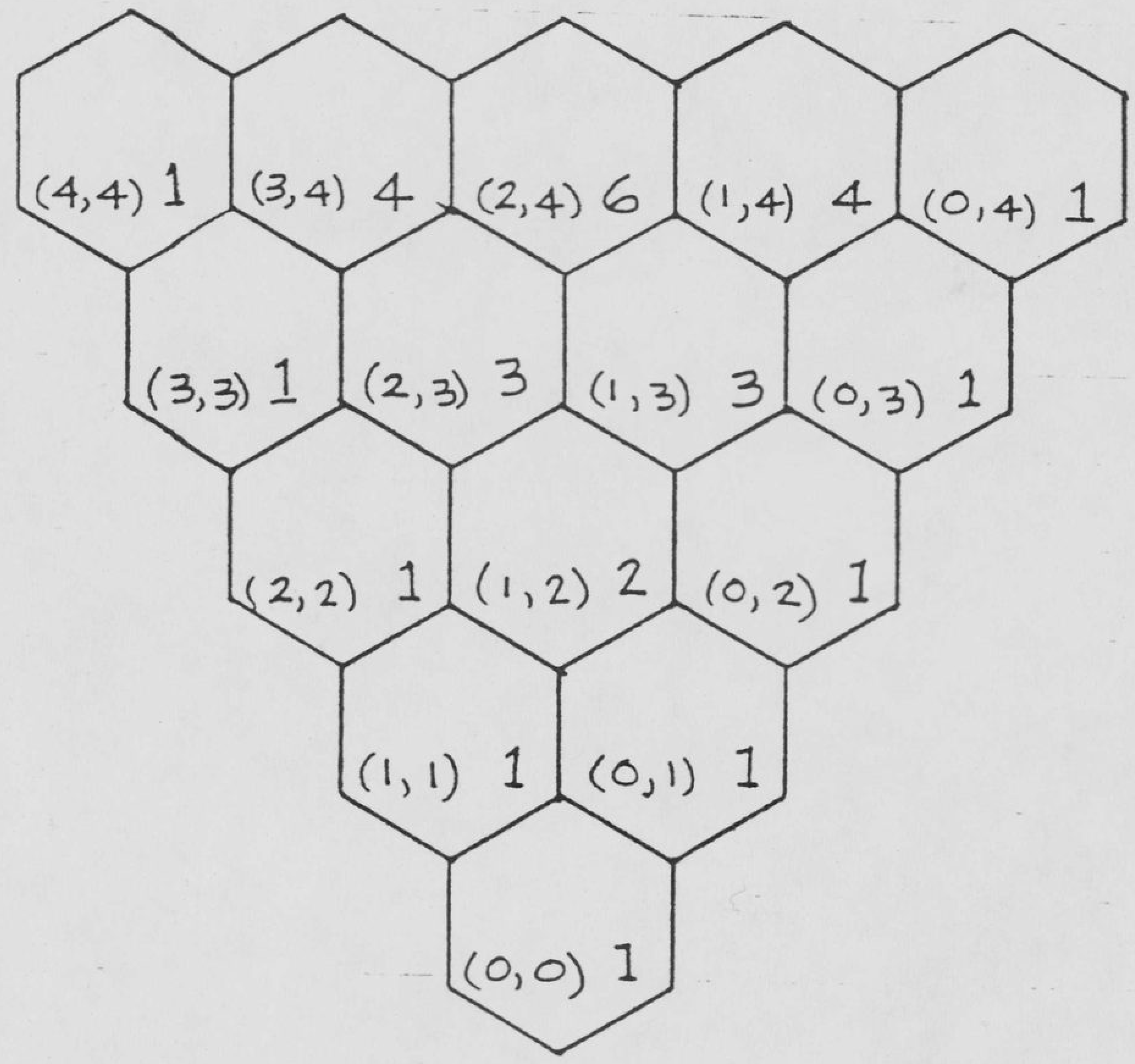


Figure 2
Pascal's Triangle rotated 180°

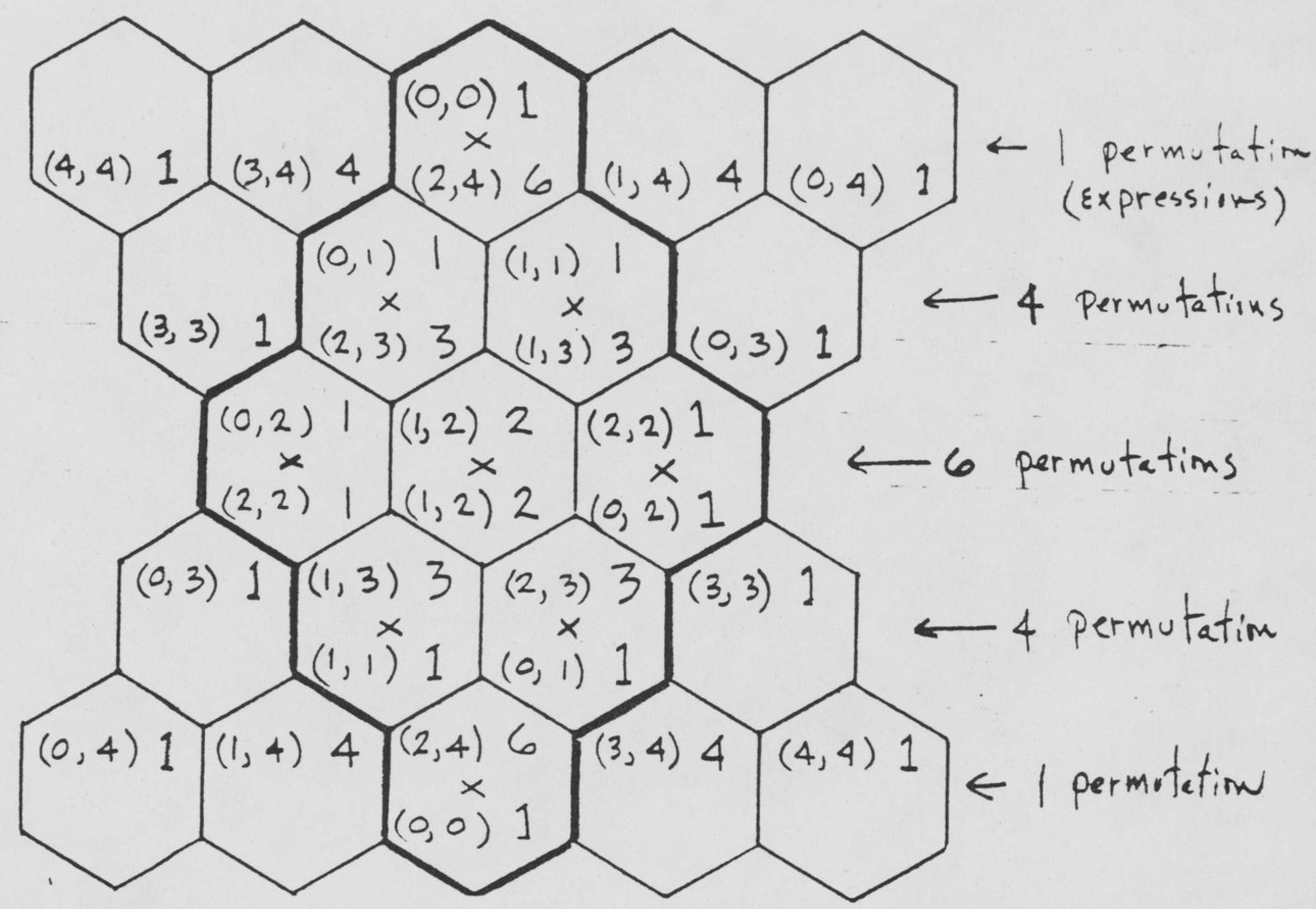


Figure 3

180° rotation of Pascals Triangle rising from the (2,4) hexany. Note; this is the key for all the partitioned cross-sets of the 2-out-of-4, (2,4) Hexany, within the outlined rhombus the cross-sets are cross-indexed, as well.

Notes on Terminology

1.

No. of Tones	Name of Combination-product Set	abbrev.	Name of Primary Module
0	———		Zeroad
1	Monany	1ny	Monad
2	Dyany	2ny	Dyad
3	Triany	3ny	Triad
4	Tetrany	4ny	Tetrad
5	Pentany	5ny	Pentad
6	Hexany	6ny	Hexad
7	Heptany	7ny	Heptad
8	Oktany	8ny	Ogdoad
9	Enneany	9ny	Ennead
10	Dekany	10ny	Dekad
15	Pentadekany	15ny	
20	Eikosany	20ny	
21	Eikosimonany	21ny	
28	Eikosioktany	28ny	
35	Triakontapentany	35ny	
70	Hebdomekontany	70ny	

2. The suffix "-ny" or "-any" is reserved for combination-product sets.
3. The "Primary Module" is a master set; that set from which the combination-product set is derived.
4. Combination-product Set; a set of combinations taken from the elements of a master set, & where the elements of each combination are multiplied. Example; the 2-out-of-4 combination-product set of the master set, 1 3 5 7, is 1·3, 1·5, 1·7, 3·5, 3·7, 5·7.