INTRODUCTION TO NONTRADITIONAL HARMONY

Ex. 1: Harmonic overtone series played once

Introduction to nontraditional harmony: tape 1, side 1

Introduction

In this demonstration, I shall build up a theory of musical harmony and consonance based on the theory of sound. I shall show how the concept of a just intonation music, in which the sound frequencies of notes forming chords lie in small whole number ratios to one another, derives naturally from observations dating back to the time of the ancient Greeks. I shall proceed to contrast a sampling of the rich resources of just intonation music with those of the equal temperament system, which is presently employed almost universally in the western world.

Although the equal temperament system provides a framework within which virtually the whole of harmonic theory is understood today, this system is itself based upon the principles of just frequency ratio music as I shall demonstrate. The resources of equal temperament music form a subset of the resources of just intonation music. In the recent past, the major thrust of effort among musicians and composers has been towards the exploitation of the very respectable resources of equal temperament music to the neglect of what lies beyond that framework.

There are three major areas in which the resources of just ratio music exceed those of equal temperament. First, chords and harmonies which are well represented in the equal temperament system have a yet purer, more harmonious sound when the note frequency ratios are closer to being exact. Second, a number of melodic and harmonic interval distinctions which arise in just ratio music based on traditional harmonies are obliterated by the equal temperament system. A music in which these distinctions are preserved is capable of greater precision and subtlety of expression than one which has lost them. Third, there are a great many chords based on frequency ratios of prime numbers greater than seven which can scarcely be represented at all within the equal temperament framework. These chords possess a unique identity and character and form a resource which has been virtually untapped until the present time.

I have built a computer-controlled music synthesizer which is capable of sounding notes having virtually any pitch within a 4½ octave range. Using this instrument, I shall demonstrate many melodic steps, harmonic intervals, chords, and chord progressions which cannot be produced with an instrument tuned to equal temperament. Where appropriate, I shall contrast just intervals and chords with their equal temperament equivalents. Thus you, the listener, will be in a position to form an opinion regarding the possibilities of a just intonation music based on first hand listening experience. Perhaps you shall come to sense as I do an excitement over the prospect that the sounds of a new and beautiful music will soon be heard.

After presenting a musical composition which employs harmonies not represented within the equal temperament system, I shall present the six harmonic intervals upon which traditional western music is based in terms of note frequency ratios. I shall show how these intervals lead to traditional chords such as major and minor triads, to a major scale, and to several different whole tone and semitone steps. Based on these, I shall demonstrate some musical problems which led to the adoption of the equal temperament system.

I shall then introduce the equal temperament approximation, pointing out some of its advantages and disadvantages. I shall contrast certain important intervals and chords by demonstrating how they sound both in just intonation and in the equal temperament representation.

Following this, I shall present the first sixteen members of a harmonic overtone series and demonstrate a number of chords which can be built from members of this series. Chords based upon lower members of the series belong to the repertoire of traditional harmonies while chords based on higher members of the series are quite new to the average listener.

Finally, I shall introduce the concept of acoustic chord inversion. This inversion is something quite different than the chord inversion of traditional theory in which the root of a chord is removed from the bass and placed in an upper part. I shall show how the process of acoustic chord inversion can generate additional new chords and demonstrate some of these chords. At various points through the demonstration I shall present chord progressions into which the new chords fit aesthetically.

At the conclusion of the demonstration I shall suggest how the technology of computer aided music synthesis can be used to exploit the riches of a just frequency ratio music.

Turning to our first example, there is one style of music which has already broken through the constraints of the equal temperament framework in certain specific ways. That style is blues. Our first example is a setting to the words of the song "Danny Boy" which I composed in blues style in order to illustrate non-equal temperament intervals and chords which are typical of that style. Two of these intervals are the septimal minor third and the blues third. The septimal minor third occurs between the fifth and the flatted seventh of a dominant seventh chord. In blues singing, the upper note of this interval is sung much flatter than it would sound played on an instrument tuned to equal temperament. The blues third lies midway between a minor third and a major third and is not represented at all in the equal temperament system. In the composition which follows, the melodic blues third is sounded at the high point of the upper melody early in the piece and it is sounded again several times towards the end.

Ex. 2: "Danny Boy"

<u>The frequency ratio basis for musical consonance</u>

It has been known since the time of Pythagoras that if a string of length l is fretted so as to leave a new length of string free which I'll call l', then there exists a special, musically satisfying pitch relationship between the tones produced when first the free string and then the fretted string is plucked if the ratio between lengths l and l' is equal to the ratio of two small whole numbers. Examples of such small whole number ratios would be 2:1, 3:2, and 4:3. The pitch interval between two tones having such a special relationship to one another is said to be a consonant interval. Each different ratio is associated with its own characteristic musical sound.

In the early 1600s, the scientists Galileo and Mersenne discovered independently that sound is carried by regular vibrations in the air and that the pitch of a sound is determined by the frequency of its vibration. Frequency is measured in cycles per second or Hertz and for most musical notes it ranges from about 50 to several thousand Hertz.

Galileo and Mersenne also discovered that the frequency of sound produced by a vibrating string is inversely proportional to its length, all other factors being the same. Thus a string whose length is fretted down to half its original length will vibrate at double its original frequency. Expressed in modern terms, two notes form a consonant interval if their frequencies lie in a small whole number ratio one to another.

The frequency ratio corresponding to the sum of two intervals is equal to the product of the frequency ratios corresponding to the individual intervals. Thus the frequency ratio corresponding to the sum of a perfect fourth (3:4) and a major third (4:5) is 3:5. This particular sum interval is known as a major sixth.

<u>In traditional</u> music <u>theory</u> all of the consonant musical intervals of western music may be expressed in terms of five fundamental intervals based on ratios of the numbers one through six. In practice, however, a sixth interval has been used implicitly in western music since the times of the renaissance. That interval is the septimal minor third based upon the ratio 6:7, which has already been mentioned in connection with the introductory piece to this demonstration.

I shall here demonstrate these six basic intervals by playing the notes forming them first in succession to form a melodic interval and then together to form a harmonic interval. For our first interval, the frequency ratio between lower and upper notes is 1:2.

Ex. 3: Octave

This interval is the octave and it is the most fundamental interval in music. For reasons still not fully understood, we perceive a certain sameness about two notes which are an octave or for that matter several octaves apart. In current music notation, notes an octave apart are given the same letter name. Also, chords in which one note has been replaced by a note one or more octaves removed from its original position are considered to be inversions of the original chord.

For the next interval, the frequency ratio of the lower note to the upper note is 2:3.

Ex. 4: Fifth

This interval is known as a fifth of perfect fifth.

For the third of our six intervals the frequency ratio is 3:4.

Ex. 5: Fourth

This interval is known as a fourth or perfect fourth. The octave, the fifth, and the fourth have been regarded as consonant intervals from the times of the ancient Greeks to the present. They were generally regarded as the only consonant intervals during most of the thousand year long medieval period.

Our next interval comprises the frequency ratio 4:5.

Ex. 6: Major Third

This interval is known as a major third.

Our next to last interval comprises the frequency ratio 5:6.

Ex. 7: Minor Third

This interval is known as a minor third or just minor third.

When notes at both a major third and a fifth above a root note are sounded together with the root note, a major triad in root position is produced. The three notes of this chord lie in the ratio 4:5:6 and I shall adopt the convention of calling the chord with these frequency ratios a four-five-six chord.

Ex. 8: Major Triad

A minor triad is formed when a root note and notes at a minor third and a fifth above it are sounded together.

Ex. 9: Minor Triad

The major and minor thirds began to appear prominently in a regional music which developed in the British Isles early in the present millenium. Over the course of some two centuries, an English style of music evolved in which these two intervals were treated as being consonant and in which triadic harmonies built from these intervals predominated. In the early 1400s, music performed by travelling English choirs aroused great admiration on the European continent. Some contemporaries wrote of being overwhelmed by its beauty. At that time the most eminent composers on the continent abandoned the style of composition to which they had been trained and made the English style their own.

It is clear that what happened is this: Those medieval British composers had seized upon two musical intervals which were naturally consonant intervals but which had been overlooked by theoreticians for a thousand years. They built a music in which these intervals formed a key ingredient, and this music had an immediate appeal to people everywhere. Is it possible that even today there exist other consonant intervals which have yet to be discovered?

The last of our six intervals comprises a frequency ratio of 6:7.

Ex. 10: Septimal Minor Third

This interval is known as the septimal minor third, the term septimal referring to the number seven. The septimal minor third may be unfamiliar to you because in the equal temperament system it is represented by the same tempered interval as that used to represent the 5:6 just minor third. To make clear the distinction between the just minor third and the septimal minor third, I shall contrast these intervals first melodically, then harmonically as two note intervals, and finally as portions of minor triadic and dominant seventh chords respectively. Following this I shall present a brief blues melody with a flatted seventh, giving the flatted seventh note a distinctive timbre.

In the first comparison, the melodic just minor third is followed by the melodic septimal minor third four times slowly and four times rapidly.

Ex. 11: Contrast melodic just minor third with septimal minor third

Next the harmonic just minor third is sounded followed by the harmonic septimal minor third five times.

Ex. 12: Contrast harmonic just minor third with septimal minor third

Now, a minor triadic chord will be followed by a dominant seventh chord. The upper note pair in the minor triad will form a minor third while the upper note pair of the dominant seventh chord forms a septimal minor third. The lower note of the upper note pair will be kept the same in each chord. The two different chords will succeed each other three times in three part harmony and three times in four part harmony.

Ex. 13: Minor triad vs dominant seventh chord

In the following brief blues style melody, I have emphasized the flatted seventh notes by giving them a woodwind timbre while all the other notes of the melody have a struck string timbre. The flatted seventh notes form septimal minor thirds with the fifth or sometimes tonic scale notes below them. I shall play the melody first at full speed and then at half speed.

Ex. 14: Blues melody

Historically there has been some controversy surrounding the septimal minor third. Mersenne, the early 17th century scientist who has already been mentioned for his discoveries in acoustics, stated that it was a consonant interval and proposed that intervals involving ratios of seven be included as integral resources of music. The French composer and theoretician Jean Phillipe Rameau, to whom modern music theory owes much, rejected this notion in a treatise published in 1726.

My own judgement and that of many others is that the dominant seventh chord, which has been heavily used in western music since the 15th century, essentially consists of four notes having frequencies in the ratio 4:5:6:7. And thus the septimal minor third, which forms the uppermost interval of this chord, is indeed one of the basic musical intervals of traditional western music, even though it has not always been consciously recognized as such.

<u>The major scale, whole tones and semitones, and the need for a tempering system</u>

A major scale can be constructed on a tonic or first note of the scale using notes forming 4:5:6 major triads on the tonic, on the note a perfect fourth above the tonic called the subdominant, and on the note a perfect fifth above the tonic called the dominant. The triads themselves are known as the tonic, subdominant, and dominant triads respectively. The scale constructed in this manner is known as the just major scale. It resembles, but is not identical with the equal temperament major scale.

I shall demonstrate how this scale is derived as follows. First I shall play the tonic, subdominant, and dominant triads in that order. Then, for each note of the scale in turn, I shall play the triad which contains the note and then sustain the scale note being illustrated after the other two notes of the triad have ceased to sound. When more than one triad contains a particular note, I shall demonstrate how the note derives from each of the two chords. Finally, I shall play the scale alone, repeating it three times.

Ex. 15: Derivation of the just major scale

The scale just played was the just major scale on F. It differs noticeably from the equal temperament major scale at four points. The whole note steps between the second and third, and between the fifth and sixth notes of the ascending scale are smaller by ten percent than the corresponding steps of the equal temperament scale. Also, the semitone intervals between the third and fourth, and again the seventh and eighth notes o f the scale are twelve percent larger than these intervals are in th e equal temperament scale. I shall play the scale again, repeated three times as before. Listen carefully for the differences I have mentioned.

Ex. 16: Just major scale repeated

In the discussion which follows, I shall speak of both the just major scales on F and on C. I shall refer to the notes of the scale on F as F, G, A, B-flat, C, D, E, and again F. As for the notes of the scale on C, I shall refer to them as C, D, E, F, G, A, B, and again C. As we shall see shortly, there is a small but important discrepancy between the pitches of the Ds in the two different scales.

Let us consider in more detail the frequency ratios between certain pairs of adjacent notes in the just major scale which we have derived.

I have already mentioned that the frequency ratio corresponding to the sum of two intervals is equal to the product of the frequency ratios corresponding to the individual intervals. A corollary to this is that the frequency ratio corresponding to the difference interval between a larger interval and a smaller interval is equal to the quotient of the frequency ratio for the larger interval divided by that for the smaller interval.

Returning to the scale on F, let us consider the frequency ratio between the tonic, F, and the second note of the scale, G. The note a perfect fifth above F is the note C. The note G lies an octave below that G which is a perfect fifth above the note C just mentioned. Adding two perfect fifths and subtracting an octave, we arrive at a frequency ratio between F and G of 8:9. Let us now consider the frequency ratio between the second and third notes of the scale, G and A. In the equal temperament scale, both intervals are identically equal. In the just major scale, the note A is a perfect major third above F and bears frequency ratio 5:4 with respect to F. Considering the interval from G to A to be the difference between the intervals F to A and F to G, the frequency ratio A to G calculates to 5/4 divided by 9/8, which is equal to 10:9. Thus in the just major scale, the steps from F to G and from G to A are unequal. The step from F to G is the larger step and it is known as a major whole tone. The step from G to A, which is about 11% smaller, is called a minor whole tone. I shall now demonstrate this difference in interval size.

First, I shall play a pattern containing only the first three notes of the just major scale, repeating the pattern four times. Listen carefully for the difference in size between the steps from the first to the second, and from the second to the third note.

Ex. 17: First three notes pattern

I shall now play a major whole tone followed by a minor whole tone on the same lower note four times, using the second note of the previous pattern as the lower note.

Ex. 18: Major vs minor whole tone on same lower note

Finally, I shall play just the upper notes of these pairs five times, beginning with the upper note of the minor whole tone step.

Ex. 19: Upper notes of whole tone steps; comma

The pitch difference between these two very slightly different notes corresponds to a frequency ratio of 80:81 and is known as a comma. This interval was known to the ancient Greeks.

I shall now demonstrate that one note, the D, of the just major scale on F is close to, but not identical with the note corresponding to it in the just major scale on C. This leads to the conclusion that a tempering or slight shifting of pitches is necessary if one wishes to make the scales on different notes superposable.

The D in the just major scale on C is the second note of the scale and, as we showed previously in the case of the just major scale on F, the second note of the scale lies a major whole tone above the tonic, having a frequency ratio to the tonic of 9:8. In the case of the scale on F, we shall determine both the interval from F to C and from F to D and subtract the first interval from the second to find the interval from C to D in this scale. The note C is a perfect fifth above F and bears frequency ratio 3:2 to F. The note D is the third in the major triad on the subdominant note, B-flat. B-flat is a perfect fourth above F and bears frequency ratio 4:3 to F. D is a major third above B-flat and bears frequency ratio 5:4 to B-flat. Multiplying ratios for the sum interval F to D, the frequency ratio of D to F is 5:3. Finally, dividing 5/3 by 3/2 to determine the difference interval between C and D, we find that the frequency ratio of D to C in the scale on F is 10:9. Thus the D of the F scale lies a minor whole tone above C.

The difference in pitch between the two Ds is small, being only a comma. One might suspect that this difference would not be very important as far as musical practice is concerned. Let us test this out by means of the following example.

Let us call the D of the C scale D and the D of the F scale D'. In the example I shall first play the pattern C D C D' twice. Then I shall harmonize the first two notes of the pattern against G and the last two notes of the pattern against F, repeating twice. The intervals formed as the harmonized pattern is played are a perfect fourth (4/3), a perfect fifth (3/2), again a perfect fifth, and finally a major sixth (5/3). Next I shall repeat the latter harmonized pattern but with a difference. When the note D is sounded in each case, I shall initially sound that version of D which gives a perfect harmony and then allow the pitch to glide to the frequency of the other D. The second interval will end as an off-fifth whose upper note is one comma or about a fifth of a semitone flat, while the fourth interval will end as a major third whose upper note is a comma sharp. Listen carefully to see how the shift in pitch affects the harmony of each interval.

Ex. 20: Test switching major and minor whole tones

Let us now listen to four part harmonizations. The pattern will first be repeated twice with the different Ds in the correct order, followed by the same pattern played twice with the Ds in reverse order.

Ex. 21: Test switching - four part harmonization

In the cases where the incorrect D was used you probably noticed a wavering beating out-of-tune sound which was particularly noticeable in the case of the off-fifth. If the D is to be usable in both the

C scale and the F scale, a tempering process must be introduced whereby the notes of neither scale form perfect triads on the tonic, subdominant, and dominant notes of the scales, but in which the worst case errors are reduced to a minimum.

We proceed to consider two different kinds of semitone. I shall play the first four notes of the just major scale on C with a chromatically altered note, an E-flat, inserted into the series. With the E-flat set at a just minor third above C, so that the frequency ratio C to E-flat is 5:6, the steps from D to E-flat, from E-flat to E, and from E to F have associated frequency ratios 15:16, 24:25, and again 15:16 respectively. Notice that the semitone step from E-flat to E is much smaller than the semitone steps from D to E-flat and from E to F.

Ex. 22: Scale steps to illustrate difference between semitones

The step between notes lying at a just minor third and at a just major third above a given lower note is called a small chromatic semitone. The interval has associated frequency ratio 24:25 and the percentage increase in frequency between lower and upper note is 4.17%. The step between notes lying at a major third and at a perfect fourth above a given lower note is known as a diatonic semitone. It has associated frequency ratio 15:16 and the increase in frequency between lower and upper notes is 6.67%. The diatonic semitone is thus 60% larger than the small chromatic semitone and the difference between the two different kinds of semitone is much greater than that between the major and minor whole tones.

I shall now contrast the diatonic and small chromatic semitones presenting first diatonic and then small chromatic semitone melodic steps from the same lower note E and following this with a two part harmonization, repeating both patterns twice. Then I shall repeat the pattern three more times using the correct semitone step initially and then gliding to the pitch corresponding to the other semitone step.

Ex. 23: Diatonic and small chromatic semitones contrasted melodically and in two part harmonization

We now test the effect of switching the two different semitones in four part harmonizations, playing the pattern twice correctly and then twice with the different semitone steps in reverse order.

Ex. 24: Diatonic and small chromatic semitones contrasted in four part harmonization

You probably noticed that the dissonances in these examples were more glaring than those in the previous ones for major and minor whole tones. The pitch errors are twice as great. Dissonances such as the ones we have just demonstrated are apt to occur when modulations to remote keys are attempted in an untempered system as for example in a modulation from C major to C-sharp major.

The last small interval which we shall consider at this point is the diesis. A diesis is the step between notes lying at a septimal minor third above a given lower note and at a just minor third above the same lower note. The frequency ratio between notes a diesis apart is 35:36 and the increase in frequency is 2.86%. Thus a diesis is slightly greater than 2/3 of a small chromatic semitone and it is roughly equal in size to an equal temperament quarter tone. I shall here play a just minor third melodic interval followed by a septimal minor third interval twice and then play only the upper notes of these intervals several times in succession. The interval between these upper notes is a diesis.

Ex. 25: Diesis

Up to this point I have demonstrated a number of interval and step distinctions which are not generally made in music today. I have omitted a number of other intervals and steps which can be derived from the six basic intervals presented thus far, such as the large chromatic semitone. I would now like to ask whether a music in which these distinctions were preserved would be worth making possible if at the same time it were also possible to prevent dissonances of the kinds demonstrated from occurring? In times past, unlike today, it was not possible to have one's cake and eat it at the same time, so to speak. A system of temperament involving some loss of the rainbow variety in sounds had to be employed in order that the basic triadic harmonies of many keys could be sounded on playable instruments.

The equal temperament approximation

I have demonstrated in the preceding section that if all note frequency ratios are held exactly, what might be called slightly differing versions of the same note will occur, with one version of the note suitable in one harmonic context and another version suitable in another harmonic context. It turns out that the more complex a piece of music is harmonically, the greater the number of notes per octave span will be required for its performance in just intonation. In some cases as many as fifty or a hundred notes within an octave span might be required.

Singers have the capability of singing any pitch whatever within their vocal range. A highly skilled sensitive group of singers can approach just ratio music performance by constantly monitoring the pitches of their voices so as to keep in tune with the harmonies being sounded by the group. The pure ratio harmonies have a "right" sound which it is naturally easy for one to zero in on. One does not need to know intellectually which D is to be used in a particular chord when singing. The right D will feel right. This is not to say that a high degree of skill and great care are not needed for the singing of music in just intonation, but at least nature is on the side of the singer in this act.

This natural advantage did not exist for performers of the lute, organ, and other fixed pitch instruments which were more and more coming into use in the 16th and 17th centuries. Compromises had to be made even at the cost of sacrificing to some extent the beauty of the harmonies sounded by these instruments. The problem of which compromises to make has challenged instrument makers and music theoreticians over the past five conturies.

In the 18th century, the equal temperament system came to prevail over other systems which were in existence at the time. It sacrifices more in the way of pitch accuracy than many of the other systems but it treats harmonies on all twelve notes of the chromatic scale uniformly. Of prime importance is that it provides a clearly recognizable representation for the five fundamental intervals of traditional music theory, namely the octave, the fifth, the fourth, the major third, and the 6:5 minor third. It reduces the complexity of harmonic theory to a level which is manageable both practically and intellectually for great numbers of people.

In the equal temperament system the octave is divided up into twelve equal semitone steps such that the frequency ratio between any two adjacent notes is the same. Expressed mathematically, this ratio is 1: the twelfth root of 2 or 1:1.059. It turns out to be a happy coincidence of the laws of mathematics that the intervals corresponding to the sums of 3, 4, 5, and 7 equal temperament semitones lie very close to the just ratios for the minor third, major third, perfect fourth, and perfect fifth respectively.

The equal temperament semitone is close to, but perceptably smaller than the diatonic semitone, and considerable larger than the small chromatic semitone.

Two equal temperament semitones make an equal temperament whole tone whose size lies between that of the minor and that of the major whole tone, being much closer to that of the major whole tone.

Three semitones in this system make an equal temperament minor third. The septimal minor third, equal temperament minor third, and just minor third have frequency ratios of 1:1.167, 1:1.189, and 1:1.200 respectively. At this point I shall compare the septimal minor third with the equal temperament minor third and follow this with comparisons of the just minor and major thirds with their equal temperament equivalents.

A major objective of this demonstration is to make clear to you just how the tempering process affects the fundamental harmonies based on the septimal, just minor, and just major thirds. In the case of the just minor and major thirds, the pitch shifts are small but clearly discernible, amounting to roughly 1/7 of a semitone. In the case of the septimal minor third, the shift is much greater, being about a third of a semitone. Because of equal temperament's almost universal acceptance today, a great many of us tend to assume implicitly that for practical purposes of music listening, an equal temperament interval is an acceptable equivalent to the just frequency ratio interval to which it corresponds. Thus we tend to selectively tune out slight variations in pitch. My personal experience is that if I am paying close attention, slight shifts in pitch due to equal temperament significantly affect the perceived character of harmonies being sounded. If I am not paying close heed, I am likely not to notice these effects. Thus you may need to listen to the following demonstrations more than once in order to gain a precise feel for the differences being demonstrated.

I shall use similar patterns to illustrate the effect of equal temperament on each of the three intervals. For the septimal minor third, I shall first sound the upper note of the interval alone at a pitch which will form a perfect 7:6 frequency ratio with the lower note of the interval. I shall then let the pitch of this note glide sharp, coming to rest at a pitch which forms an equal temperament minor third with the lower note of the interval. The pitch will again glide flat to its initial frequency, cycling between its just septimal and equal temperament values three times before the lower note of the interval begins to sound just as the pitch of the upper note glides to rest at a perfect septimal minor third above the lower note. With the lower note now sounding at a steady pitch, the upper note will glide through three more cycles with the last interval sounded being an equal temperament minor third.

The pattern will then be recommenced with a different timbre, and additional notes forming a 4:5:6:7 dominant seventh chord with the upper note will chime in one by one. All the while the upper note will slowly alternate between pitches at a septimal minor third and at an equal temperament minor third above the next highest note. The continuous pattern will end with the upper interval forming an equal temperament minor third. Finally, a just 4-5-6-7 chord will be sounded.

Ex. 26: Contrast of septimal minor third with equal temperament minor third

In the following example I shall contrast the just minor third with the equal temperament minor third. Again the upper note of the interval will alternate between the pitches corresponding to the two different intervals while all the other notes hold their pitches constant. Here, the pitch corresponding to the just minor third is higher than that corresponding to the equal temperament minor third. In the second part of the example I shall sound the second inversion, the root position, and the first inversion of the minor triad in that order as the note a minor third above the root note oscillates between its just and equal temperament pitch values. In this way you will be able to hear the effect of tempering the interval on all three inversions of the minor triad. As before, the continuous pattern ends on an equal temperament chord while the separate chord played at the very end is a just minor triad.

Ex. 27: Contrast of just minor third with equal temperament minor third

Proceding to the major third, four equal temperament semitones make an equal temperament major third, which is slightly larger than the just major third. The frequency ratios are 1:1.250 for the just interval and 1:1.260 for the equal temperament interval. In the following example I shall compare the intervals as for the previous two examples, testing different inversions of major triads in the second part of the example.

Ex. 28: Contrast of just major third with equal temperament major third

We have compared the three just third intervals with their equal temperament equivalents and heard triadic and dominant seventh chord harmonies with both just and tempered thirds. I would like to ask at this point: "How does tempering the intervals affect the harmonies of the chords in each case? Is the effect of temperament for all practical purposes negligible, or, on the other hand, does it significantly alter that subtle emotional flavor which the various harmonies carry with them?" My own experience is that the harmonies are significantly altered by temperament in each case. The major triads in particular have to my ears a sweet harmonious sound in just intonation which they lose when the major third is sharped to its equal temperament value.

Proceeding to the larger intervals, five equal temperament semitones come very close to making a perfect fourth and seven equal temperament semitones come equally close to making a perfect fifth. The frequency ratio for an equal temperament fifth is 1:1.498 as compared with 1:1.500 for the perfect fifth. The equal temperament errors in the intervals of the fourth and fifth are only about 1/7 as great as the errors in the major and minor thirds. Finally, by definition twelve equal temperament semitones form a perfect octave.

Over the past two centuries or so, the equal temperament system has provided the western world with a highly practical usable theoretical framework for music based upon five of the six fundamental consonant intervals of traditional music. Even the sixth interval, the septimal minor third, is considered by many to be passably represented by the equal temperament minor third. This theoretical framework serves as the basis for understanding music which is presently taught in schools and used by most composers and performers.

I have already asked whether incorporation of those melodic and harmonic step distinctions which are obliterated in the equal temperament system, such as the one between chromatic and diatonic semitones, would enrich musical art. I have also questioned whether tempering the various just third intervals significantly alters the harmonies built on them. I would now like to pose this question: "Do intervals and chords based on frequency ratios involving higher prime numbers than seven, numbers such as 11 and 13, have something new and unique to offer to music?" I suggest that they do, and I shall demonstrate a number of intervals and chords based upon these numbers.

The overtone series

I shall introduce what may be called the more remote harmonies based upon ratios of 11 and 13 in a logical progression based upon natural laws of acoustics. I shall show how they are derived by an extension of the same process by which the familiar triadic and seventh harmonies are derived. I shall do this using the harmonic overtone series.

It was the early 17th century investigator Mersenne, twice mentioned already, who observed that when a note is sounded by a trumpet, a whole series of tones could be discerned at pitches lying above that of the note being sounded. These overtones lie at an octave, an octave and a fifth, two octaves, two octaves and a major third, and so forth above what is called the fundamental. The frequencies of the overtones lie at twice, three times, on up to n times the fundamental frequency and beyond, where n is an integer. Mersenne's work led to the discovery that virtually all musical tones are actually composite vibrations consisting of the sum of a series of sinusoidal vibrations at the fundamental frequency and at 2, 3, ... n times the fundamental frequency and so forth. In general, the amplitude of the overtones decreases as n becomes very large. Each musical instrument produces sounds with characteristic overtone patterns in which different harmonics have different relative strengths. In fact, the overtone pattern of a sound has a major influence on the sound's quality or timbre.

I shall here demonstrate a harmonic series consisting of a note at a fundamental frequency followed by notes at the frequencies of the second, third, fourth, etc. on up to the sixteenth harmonic while the note at the fundamental frequency is sustained. I shall present the series twice. Note in particular the 11th and 13th members, which I have prolonged and given distinctive amplitude envelopes.

Ex. 29: Harmonic overtone series

Because of the way in which the harmonic overtone series is constructed, any chord consisting of notes which are all members of the series must have its note frequencies lying in small whole number ratios to one another. It turns out that chords based on the lower members of such a series are the chords of traditional music while chords based on higher members of the series have been little heard or appreciated, but yet possess that uniqueness and harmoniousness which characterize the more traditional chords.

Starting with lower members of the series and proceeding up to higher members, the four note chord consisting of the very first four members of the series is the open fifth chord.

Ex. 30: Open fifth chord

It is a chord possessing great stability and a sense of finality. A great many works of the renaissance period came to their final resting point on this chord.

The second, third, fourth, and fifth members of the series give us a major triad in root position. Following the previously introduced convention, I shall call this chord the 2-3-4-5 chord.

Ex. 31: 2-3-4-5 chord

The 4-5-6 chord, which has already been demonstrated, is also a triad in root position.

Ex. 32: 4-5-6 chord

Adding the 7th member of the harmonic series to this last chord yields a dominant seventh chord.

Ex. 33: 4-5-6-7 chord

The 4-5-6-7-9 chord is a dominant ninth chord.

Ex. 34: 4-5-6-7-9 chord

The upper two notes of this chord form a 7:9 frequency ratio interval which is decidedly larger than the major third and is not treated in traditional music theory based on the equal temperament system. I shall contrast the 7:9 interval with the just major third, beginning with the major third and presenting three comparisons.

Ex. 35: Just major third vs 7:9 interval

The 6-7-9 triad forming the upper three notes of the dominant ninth chord somewhat resembles a minor triad. The frequencies of the notes of the minor triad are in the ratio 10:12:15 or 6:7.2:9. I shall play pairs of these chords starting with the just minor triad and presenting three chord pairs.

Ex. 36: Just minor triad vs 6-7-9 chord

Let us compare alternative supertonic to tonic resolutions using first the supertonic minor triad and then the 6-7-9 chord. After the first two comparisons, I shall add a fourth note at a just minor third below the lower note of the first triad in each resolution. Both resolutions strike me as being satisfying, but they are subtly different.

Ex. 37: Alternative supertonic to tonic resolutions

<u>The first chord</u> in this series which in my opinion differs strikingly from the chords of traditional music is the 7-9-11 triad.

Ex. 38: 7-9-11 triad

The six note 4-5-6-7-9-11 chord, which might be called a dominant eleventh chord sounds thus.

Ex. 39: 4-5-6-7-9-11 chord

Consider the triads formed by members 6, 8, and 10, 7,9, and 11, and 8, 10, and 12 of the harmonic overtone series. The first and third chords are major triads in second inversion and root position respectively. The 7-9-11 chord fits beautifully as an intermediate step between these chords. The following example illustrates this and then proceeds on to other inversions of the progression.

Ex. 40: Chord progression using the 7-9-11 triad (1)

At many points in the preceding example, the melodic steps taken by the three voices cut boldly between the usual choices of semitone or whole tone and move through 10:11 and 11:12 frequency ratio intervals.

The 7-9-11 chord may also act as an intermediate between two major triads in a different way. Here, 4-5-6, 7-9-11, and 3-4-5 triads are built on the same middle note in succession. I shall play the series of chords 4-5-6, 3-4-5, and 2-3-4, all with the same middle note and follow this by the same series with the 7-9-11 chord inserted between the first two chords of the original series, repeating the first series twice and the second three times.

Ex. 41: Chord progression using the 7-9-11 triad (2)

The blues third lies intermediate between the minor and major third intervals and I believe it to be the 9:11 frequency ratio interval. I shall here contrast the just minor, blues, and just major thirds in that order repeating three times.

Ex. 42: Just minor, blues (9:11), and just major thirds contrasted

The chord most closely resembling the 7-9-11 triad in the equal temperament system is the augmented triad. Let us compare these two chords, playing the 7-9-11 chord followed by the equal temperament augmented triad with the same upper note four times.

Ex. 43: 7-9-11 triad vs equal temperament augmented triad

My personal opinion is that while the 7-9-11 chord has a very distinctive eerie sound, the augmented triad is a much tamer chord.

Proceeding to chords involving the thirteenth member of the harmonic overtone series, the 9-11-13 chord sounds thus.

Ex. 44: 9-11-13 triad

Here is the six note 4-6-7-9-11-13 chord.

Ex. 45: 4-6-7-9-11-13 chord

The 9-11-13 triad is intermediate between the harmonic diminished triad having notes with frequencies in the ratio 5:6:7 and the 4-5-6 major triad. I shall play the 5-6-7, 9-11-13, and 4-5-6 chords on the same lower note in a cycling succession and follow this by a four part harmonization with an added lower note consisting of the 4-5-6-7, 7-9-11-13, and 3-4-5-6 chords played in the same pattern. The lower three notes of the last chord group follow a pattern which was presented in the preceding section.

Ex. 46: 5-6-7, 9-11-13, 4-5-6, 9-11-13, ... chord progression followed by 4-5-6-7, 7-9-11-13, 3-4-5-6, 7-9-11-13... chord progression

Finally, we introduce the 9-11-13-15 chord.

Ex. 47: 9-11-13-15 chord

Playing the 9-11-13-15 chord followed by the 8-10-12-16 chord on the same fundamental note yields an alternative to the traditional dominant seventh to tonic resolution. Let us compare the two.

Ex. 48: 9-11-13-15 to 8-10-12-16 (4-5-6-8) resolution followed by dominant seventh to tonic resolution

Members 8 through 16 of the harmonic overtone series form an eight tone octave scale which is in some ways analogous to, but quite different than the 12 tone scale of equal temperament. Here is how it sounds.

Ex. 49: Eight tone scale based on harmonic overtone series

This scale has some significant characteristics. The most important one is that any chord whatsoever which is formed from notes taken from it will have frequency ratios which can be expressed in terms of whole numbers equal to fifteen or less. Thus these chords will all possess a degree of consonance. The chords which have been demonstrated here form only a fraction of the some two hundred odd unique chords having three or more notes which may be derived from this scale and thorough exploration of its harmonic potential should prove to be an exciting and rewarding endeavor for composers.

I shall now demonstrate a chord progression based on this scale which in a way recapitulates what we have covered in this section. Following this, I shall introduce the concept of scale and chord inversion and show how a great many more chords still may be derived by inverting the harmonic overtone series and building up chords from the inverted series in the same manner as that used to construct them from the original series.

The chord progression based on the scale is derived as follows. A chord is formed on each note of the scale consisting of the note itself plus the second, fourth, and sixth lower members of the harmonic overtone series below it. Thus the chord on the tonic, which is number eight of the series, consists of members eight, six, four, and two of the series. Here is the complete progression ascending and descending the scale several times.

Ex. 50: Chord progression based on harmonic overtone series

<u>Expansion of the repertoire of harmonies through chord inversion</u>

Let us consider the 4-5-6 major triad in root position.

Ex. 51: Root position major triad

This chord consists of a major third on the lowest note, above which is a minor third.

Ex. 52: Major third on root with minor third above it giving a root position major triad

We may derive another chord from this one using the following procedure. Take the root note, build a major third below it, and then build a minor third below the major third giving what I'll call the acoustic inversion of the first chord.

Ex. 53: Construct acoustic inversion of major triad

The resulting chord, a minor triad, is distinctly different than the first chord even though it was derived very simply from the major triad using the same intervals as those which constitute the major triad with only the order of those intervals reversed. Because the inversion of a chord contains the same intervals as the original chord, if all the intervals of the original chord are consonant, so will the intervals of the inverted chord be consonant.

Every chord based on the harmonic overtone series which has been presented thus far possesses an acoustic inversion which, in the cases we have looked at, is different than the original chord. Not only does each chord possess an acoustic inversion, but so also does every melodic sequence including the harmonic overtone series itself and the scale based upon it.

I shall here present the inversion of the harmonic overtone series, which I shall call the undertone series. I shall then present the inversions of some familiar chords based upon lower members of the harmonic overtone series. Following this, I shall present inversions of some of the nontraditional chords which we have heard. This shall lead us to the conclusion of our demonstration.

Here is the undertone series.

Ex. 54: Undertone series repeated twice

The open fifth and the 3-4-5 and 4-5-6 forms of the major triad invert as follows.

Ex. 55: 2-3-4, 3-4-5, and 4-5-6 chords followed by their inversions

The 4-5-6-7 dominant seventh chord inverts to a diminished minor seventh chord.

Ex. 56: 4-5-6-7 chord and its inversion

If a chord sequence for a dominant seventh to tonic resolution is inverted in its entirety, what is known in music theory as a supertonic to tonic resolution in a minor key results.

Ex. 57: 4-5-6-7 to tonic resolution and its inverse

The 7-9-11 chord inverts as follows.

Ex. 58: 7-9-11 chord followed by its inverse

Here follows the 6-8-10, 7-9-11, 8-10-12 chord progression on the same fundamental alternating with its inversion.

Ex. 59: 6-8-10, 7-9-11, 8-10-12 chord progression followed by its inversion

Here is the 9-11-13 triad followed by its inversion.

Ex. 60: 9-11-13 triad and its inversion

Again I shall repeat an earlier presented progression, this time for the 7-9-11-13 chord, followed by the inversion of that progression.

Ex. 61: 4-5-6-7, 7-9-11-13, 3-4-5-6, 7-9-11-13 progression on same next to lowest note followed by the inversion of the progression

Arriving at the 9-11-13-15 chord, this chord and its inversion sound thus.

Ex. 62: 9-11-13-15 chord and its inversion

Finally, I shall present the inversion of the scale built from the 8th through 16th members of the overtone series followed by the inversion of the previously presented chord progression based on this scale.

Ex. 63: Inverted eight tone scale

Ex. 64: Chord progression based on inverted scale

Conclusion

In this demonstration, I have outlined the foundation upon which a just frequency ratio music is constructed and presented the musical intervals which constitute this foundation. I have explained how the equal temperament approximation is derived from just frequency ratio music by dividing the octave into twelve equal steps and representing the fundamental intervals of music by fixed numbers of steps - 12 for the octave, 7 for the fifth, 5 for the fourth, 4 for the major third, and 3 for the minor third, as you will recall.

I have shown that while the equal temperament system represents these intervals surprisingly well, the resources of just ratio music exceed those of the equal temperament subset in three important respects. These are that traditional chords have a yet more harmonious sound in just intonation, that the additional step and interval distinctions of just frequency ratio music give it the potential for greater variety and precision of expression than has equal temperament music, and that, perhaps even more significantly, a great many chords and harmonies are available to just intonation music which have no representation at all within the equal temperament framework.

I have contrasted intervals and chords as represented in the two systems and introduced a number of melodic steps and harmonic intervals lying outside the equal temperament framework. I then presented a number of new chords and chord progressions based upon note frequency ratios involving the numbers eleven and thirteen.

It often happens that a knotty problem will remain unsolved for a long time. Then it is solved, but only after the framework of assumptions and rules initially used to approach the problem has itself been put aside and an entirely fresh attack on the problem has been made. A question which many composers have asked themselves has been this: " How can I make a statement in music which has not already been made before? " This demonstration provides an answer to that question. It is this: " Free yourself from the constraints of equal temperament and make bold use of the full melodic and harmonic resources of just intonation music. "

There is a snag here. The reflective listener may respond: " It is all very well to suggest using these resources. I believe they exist. But who is going to perform this new music? How is a performer going to manage fifty or more different notes per octave span with only two hands and at the same time play six or eight part harmony at normal speed? "

Twenty, or even ten years ago there would have been no adequate answer to these questions. Today, all the technology is there to allow a human performer at a keyboard to perform music in just intonation which is as full and intricate as that which he or she can now play in equal temperament tuning. The microprocessor, when incorporated into the pitch control circuitry of a keyboard synthesizer or electronic organ can make this possible.

I shall conclude with an appeal to the instrument designer. Using a relatively inexpensive home built digital synthesizer controlled by a home microcomputer, I have demonstrated some of the melodic and harmonic possibilities of a music unconstrained by an equal temperament tuning system. The research and development effort required to produce a marketable keyboard capable of playing just intonation music is not excessive and the potential rewards are very great. The capabilities and limitations of human performers provide the baseline standard to which the instrument must be designed. Once it has been decided by means of which controls the performer will play it, the software and hardware for fine control of the note pitches may be developed. The software engineer in particular must have a solid grasp of the principles of harmony in addition to the usual programming skills. Might it not be a very sound move to design, to promote, and ultimately to market a just intonation keyboard instrument?

Thank you for sharing with me this listening adventure into the less well travelled regions of the universe of music.