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RESEARCH NOTES

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When investigating a tuning for which there is no
repertoire or tradition of any kind, the most illuminating
approach is to look for connections between the new
tuning and 12-note equal. All the other tunings contain many
extremely discordant intervals, and are amenable to
row music, or to other non-tonal compositional techniques. It
But the most interesting are those that include tonal
elements — i.e, major and minor triads, and seventh chords,
which may be arranged in ways similar to, but not
exactly like, 12-note equal. There are two simple tests that
reveal some of those connections: One is whether or not the
new tuning contains recognizable diatonic scales, and the
other is whether or not it contains symmetric modes.

By "recognizable diatonic scale," I mean a tuning
of the notes C, D, E, F, G, A, B, C such that CD, DE,
FG, GA, and AB are all equal, and larger than EF and
BC, these last two being also equal. This insures that
the scale may be evolved from the broken circle of
fifths F C G D A E B F (BF is diminished, the others are
perfect and equal) in the same manner as in Pythagorean
tuning (fifths are pure), meantone tuning (major thirds are
pure) and 12-note equal tuning (something in between the
other two). There is a simple arithmetical principle
which determines whether or not a given equal tuning

does or does not contain recognizable diatonic scales. The tuning contains such scales if, and only if, the total number of notes may be expressed as $5W+2H$, where W and H are integers such that $0<H<W$. (This is rigorously demonstrated in Book #1, which is near completion.) If we apply this test to the tunings of up to 24 notes, we find that only five of these contain recognizable diatonic scales. Clearly the least possible values for H and W under the conditions as stated are $H=1$, $W=2$, and then $5W+2H = 12$ (which should come as no surprise.) If $H=1$, $W=3$, we have $5W+2H = 17$; if $H=2$, $W=3$, we have $5W+2H=19$; if $H=1$, $W=4$ $5W+2H=22$; if $H=2$ $W=4$ $5W+2H=24$. In all other cases where $0<H<W$, it is also true that $5W+2H > 24$.

A symmetric mode is a division of an octave into an even number of intervals, with two interval sizes occurring alternately. To illustrate, two such modes (plus their transpositions) occur in 12-note equal tuning: the ones of eight notes and six notes. These make the following arrangements relative to C, with the larger interval occurring first:

positions	0	1	2	3	4	5	6	7	8	9	10	11
notes	C		D	Eb		F	F#		G#	A		B

8-note symmetric mode

positions	0	1	2	3	4	5	6	7	8	9	10	11	12
notes	C			Eb	E			G	Ab			B	C

6-note symmetric mode

Examples of passages in these modes are frequent throughout the first half of the 20th century. For the 6-note mode, see Bartok Concerto for Orchestra, 3rd movement bars 10 – 27, where several transpositions are used (the oboe melody is freer). Also see Hindemith Herodiad, the 21 bars beginning at letter (J) (there are free notes in the melodies). Uses of the 8-note mode are much more frequent, Examples:

Bartok - Music for Strings, Percussion, Celesta, 2nd movement bars 185-201

Bartok – Fourth Quartet, 5th movement, bars 12-36 (somewhat free use – Ab replaces Eb)

Stravinsky – Sacre du Printemps, number [42] to [43], and again [44] to [45] (strict use), also [131] – [133].

Stravinsky – Les Noces, (82) – (87) (strict)

Stravinsky – Symphony in 3 Movements, numbers 7 – 13

Messiaen – Practically everything from the 1940's. (This composer's attachment to the 8-note mode reaches the intensity of a fetish.)

The interesting feature of these modes, aside from their historical use, lies in the fact that they

$\sqrt[5]{2}$

$\sqrt[7]{2}$

the 6-note mode: 12, 15, 18, 21, and 24 notes; and the following contain the 8-note mode: 12, 16, 20, and 24 not

Now comes the interesting part: According to the definitio there must exist a symmetric mode of 10 notes, quite different from what is familiar to trained musicians, but related somehow to the other symmetric modes, and the relations they establish. Using the same test, this 10-note mode must exist within the equal tunings of 15 and 20 notes. By the same considerations, 18-note equal tuning contains a 12-note symmetric mode, and 21-note equal tuning contains a 14-note symmetric mode.

It is also of interest to determine which equal tunings contain what I call "nearly just" tunings. A nearly just tuning is one which arranges versions of major tones, minor tones, and semitones, and preserving their relative sizes, as is found in true just tuning. Call the major tone M, minor tone N, and semitone S; the well-known arrangement is

ratios		$\frac{9}{8}$		$\frac{10}{9}$		$\frac{16}{15}$		$\frac{9}{8}$		$\frac{10}{9}$		$\frac{9}{8}$		$\frac{16}{15}$
notes	C		D		E		F		G		A		B		C
intervals		M		N		S		M		N		M		S

an arrangement of 3 major tones, 2 minor tones, and 2 semitones. The number of notes in a nearly just equal tuning is then $3M + 2N + 2S$, where M, N, and S are integers such that $0 < S < N < M$.

Now if $S = 1$, $N = 2$, and $M = 3$, we have $3M + 2N + 2S =$ 1
if $S = 1$, $N = 2$, and $M = 4$, we have $3M + 2N + 2S = 18$;
if $S = 1$, $N = 3$, and $M = 4$, we have $3M + 2N + 2S = 20$; and
if $S = 2$, $N = 3$, and $M = 4$, we have $3M + 2N + 2S = 22$. (This 1 tuning and some of its nearly just properties have been discussed by other theorists.)

More light on these tunings comes from determining the size of their perfect fifths and major thirds. If we call an octave a, the perfect fifth is equal to $\frac{2M+N+S}{3M+2N+2S}a$ and a major third to $\frac{M+N}{3M+2N+S}a$. For example, in 22-note equal tuning, a perfect fifth is $\frac{13}{22}a = 709.091$ cents, and a major third is $\frac{7}{22}a = 381.818$ cents. Similar data for the other tunings are as follows:

Note—
pure perfect fifth 701.955 cents
pure major third 386.314 cents

number of notes	15	18	20	22
perfect fifth (cents)	$\frac{9}{15}a = 720.000$	$\frac{11}{18}a = 733.333$	$\frac{12}{20}a = 720.000$	$\frac{13}{22}a = 709.09$
major third (cents)	$\frac{5}{15}a = 400.000$	$\frac{6}{18}a = 400.000$	$\frac{7}{20}a = 420.000$	$\frac{7}{22}a = 381.818$

A quick study of the above indicates that only the equal tuning of 22 notes can be realistically called "nearly just." If we take middle C to be 264 Hz. (I recommend this as the standard value), the fifth beats at $264(2 \cdot 2^{13/22} - 3) = 3.271$ per second, and the major third at $264(4 \cdot 2^{7/22} - 5) = -3.423$ per second. In the case of 15 notes, the major third of 400 cents is the same as that interval in 12-note equal tuning. Perfect fifth middle CG beats at $264(2 \cdot 2^{9/15} - 3) = 8.298$ per second — a version of this interval that is very distinctly out of tune. But the effect is no more disturbing than what is deliberately introduced into the organ in the form of celeste ranks. At the present time I conclude that the 15-note equal trial, while not "nearly just," does indeed have harmonic values that are not unpleasant. However the 733.333 cent perfect fifth associated with 18-note equal tuning is altogether too large, and the same may also be said of the 420 cent major third of 20-note equal tuning. In general, major thirds that exceed the Pythagorean value of 407.820 cents are not euphonious harmonic combinations. This consideration tends to make the diatonic scales contained by 17 and 22 note tuning undesirable. In the former case, the major third is equal to $\frac{6}{17}a = 423.529$ cents; in the latter case $\frac{8}{22}a = 436.364$ cents. (I do not want to reject these out of hand without considerable experimentation on the Scalatron.)

It is useful to sum up the harmonic possibilities of each tuning, so far described, in the following chart:

Number of notes	Recognizable diatonic scales	Nearly just diatonic scales	symmetric modes: 6-note	8-note	10-note	12-note	14-note
12	yes	no	yes	yes	no	no	no
13	no	no	no	no	no	no	no
14	no	no	no	no	no	no	no
15	no	yes	yes	no	yes	no	no
16	no	no	no	yes	no	no	no
17	yes	no	no	no	no	no	no
18	no	yes	yes	no	no	yes	no
19	yes	no	no	no	no	no	no
20	no	yes	no	yes	yes	no	no
21	no	no	yes	no	no	no	yes
22	yes	yes	no	no	no	no	no
23	no	no	no	no	no	no	no
24	yes	no	yes	yes	no	yes	no

From this it appears that the most alien tunings are those of 13, 14, and 23 notes. [Although I am not sure yet, there may be other types of organization which would open other possibilities for 16, 20, and 24 notes, as well as yet another which would affect 14 notes. This will require many hours of listening.]

$\sqrt[5]{1/6}$ $\sqrt[7]{1/6}$ $\sqrt[14]{6}$ $\sqrt[23]{2}$

5 sheep

7 goats

$\sqrt[5]{\ }$ $\sqrt[7]{\ }$ $\approx \sqrt[17]{2}$ $\approx \sqrt[23]{3}$

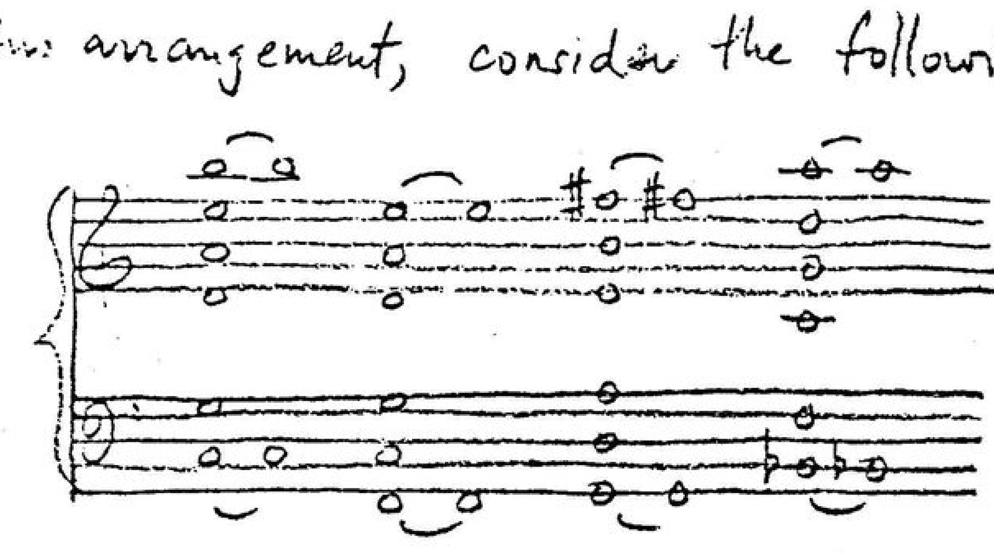
I have decided to focus first on 15-note equal tuning; first because the number of notes is relatively small; second because it seems that the 10-note symmetric mode offers many possibilities of new harmonic organization of triads.

We now come face to face with the problem of devising a comprehensible notation. I am firmly of the opinion that any notation is worse than useless if it is not compatible with the existing notation, and with musical habits. In view of this, I think it essential to retain the five-line staff, along with the conventional clefs. Also desirable is the use of the # and b, essentially with their usual meanings, with perhaps a new accidental whose operation may be simply defined. [I must admit that I was badly stuck on this point until about a month ago, when suddenly everything broke through clearly.]

What was wanted was an organization in which triads look like triads, essentially as we are accustomed to seeing them. Equally important is the comprehensible depiction of the division of the octave into five equal parts, and the 10-note symmetric modal arrangements. All this should be accomplished without hiding the existence of the 6-note symmetric modes, or augmented triads.

A suggestion as to how to proceed comes from the fact that the sum of five 720-cent fifths is exactly equal to

three octaves, from the relation $5 \times 720 = 3 \times 1200 = 3600$. In this arrangement, consider the following:

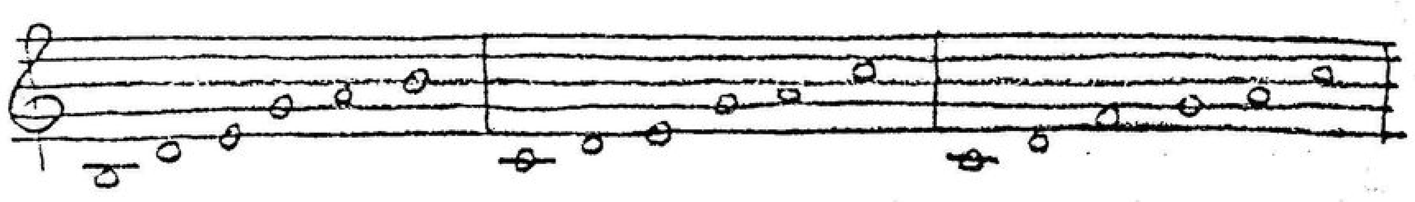


1. 2. 3. 4.

In each case the outer pitches are separated by three octaves exactly, and it thus appears that BC, EF, F#G, and ABb are "enharmonically equivalent," to borrow an ancient, imprecise phrase. In reality, this is no stranger than what is normally associated with AbG#, DbC#, EbD#, etc. (Musicians tend to forget that much of what they take for granted seemed very perplexing, prior to being learned by rote.)

In order to get an idea of the sound of the 5-note equal division, with each interval equal to 240 cents, we note first that this is larger than a major second, and smaller than a minor third. To relate this to the notation described above, consider a major scale first in Pythagorean tuning, and then imagine the major seconds gradually increasing. As the five equal major seconds each increase by 2 cents, both minor seconds decrease by 5 cents. Eventually there must come a point where the two minor seconds disappear entirely, leaving only the five major seconds, each equal to 240 cents, and this is a 5-note equal division. Clearly a scale

C D E F G A B C where EF coincide, as do BC, is not in any sense a "recognizable diatonic scale"; but it does relate the 5-note equal division to the arrangements of p. 11, and illustrates the generality of the disappearance of the minor second. With this in view, it may be seen that all the configurations below are, in fact, divisions of octave middle C-C next above into five equal parts.



To read these arrangements with conventional musical habits gives a somewhat inaccurate impression of their true sound, which is not, however, entirely off the mark. They sound, in fact, like out-of-tune pentatonic scales, in which the major seconds and minor thirds have become equal, and whose distribution is uncertain as a consequence.

The notation of the 10-note symmetric modes, and of the entire 15-note equal tuning, may be completed by introduction of two new accidentals [These will undoubtedly have application to some of the other tunings as well.] I have hit on the symbols ↑ and ↓; the circles aid in making it clear which note in a chord is affected, by placing the accidental

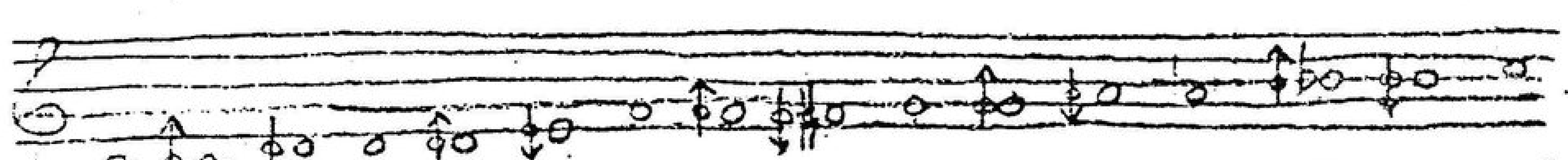
unmistakably on a line or in a space. I call these accidentals up and down (why use complex words for a simple concept?). Accordingly D↓ is read "D-down," F#↑ is read "F-sharp-up," etc. It is also true that F#↑ is the same as G↑, D↓ is the same as Eb↓, etc. Using this scheme, one notation for a 10-note symmetric mode is

C, D↓, D, E↓, E, G↓, G, A↓, A, C↓, C. notes
0 2 3 5 6 8 9 11 12 14 15 positions

Equally correct is the following (F replaces E; F#↓ = G↓, B↓ = C↓)

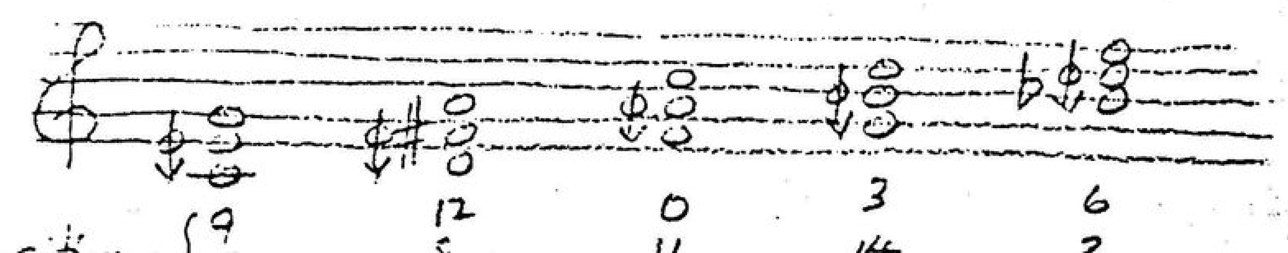
C, D↓, D, E↓, F, F#↓, G, A↓, A, B↓, C notes
0 2 3 5 6 8 9 11 12 14 15 positions

Note that ↓ occurs in both arrangements, but not ↑.
The whole 15-note equal tuning may be notated as follows:



0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15

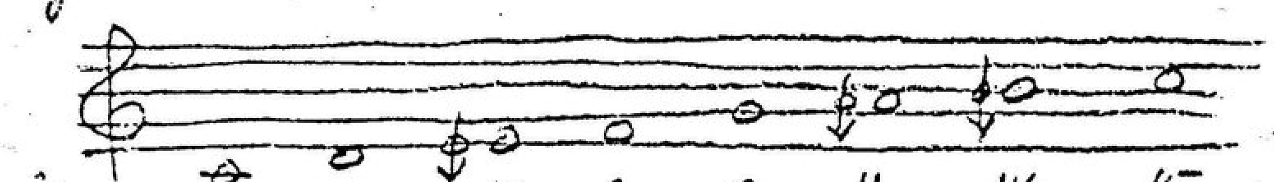
If we take a major triad to contain a major third of 400 cents ($\frac{5}{15}a$) and a perfect fifth of 720 cents ($\frac{9}{15}a$), as in the nearly just configuration, we see that the triad whose root is C is spelled C, E↓, G. This triad, along with those whose roots are D, F, G, and Bb, appear as follows, remembering that A and Bb are the same:



positions	9	12	0	3	6
	5	8	11	14	2
	0	3	6	9	12

It will be observed that these five triads, whose roots divide the octave into five equal parts, are contained by the 10-note symmetric mode in positions 0, 2, 3, 5, 6, 8, 9, 11, 12, 14.

If we construe CEɸG as the tonic, FAɸC as the subdominant, and GBɸD as the dominant, we have the following scale:



positions	0	3	5	6	9	11	14	15	
differences	3	2	1	3	2	3	1		(sum = 15)

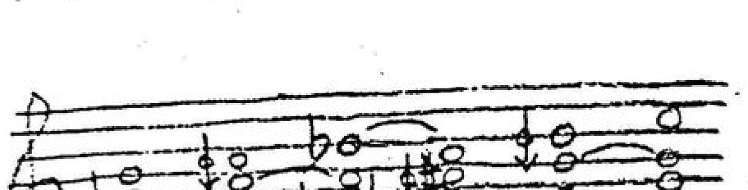
Now in the "difference" line, replace 3 by M, 2 by N, and 1 by S to obtain M, N, S, M, N, M, S. But this is the arrangement of major tones (M), minor tones (N), and semitones (S) that make up a nearly just scale (see p. 6). Hence <u>the nearly just configuration of 15-note equal tuning is contained by one of the 10-note symmetric modes.</u> This establishes a connection between diatonic and chromatic harmony utterly unlike anything in 12-note equal tuning, and is, I think, a discovery of major importance.

Much more can be said about the nearly just configuration by calling upon the known properties of true just tuning. In particular, fifth DA in true just tuning is in the ratio $\frac{40}{27}$, short from pure (701.955 cents) by a syntonic comma (21.506 cents). In 15-note equal tuning, this fifth is notated DAɸ, distinguishing it at a glance from the other perfect fifths. (The broken circle appears as F, C, G, D, Aɸ, Eɸ, Bɸ.) This "fifth" is equal to $\frac{8}{15}a$ = 640 cents, grossly exaggerating the principal defect of true just tuning. The minor triad including F and Aɸ is better tuned DɸFAɸ; but I do not like (nor can I find anyone who does) the effect of Dɸ in II coming right next to D in V, such as the following:

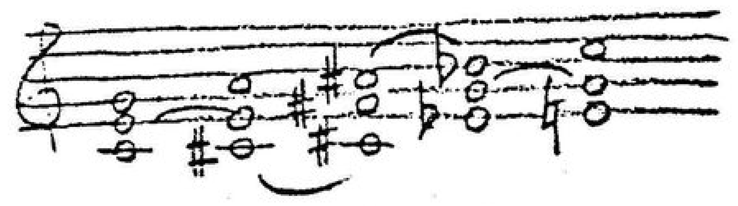
I VI II V I

For this reason, I am inclined, at the present time, to recommend that the II–V progression should be avoided.

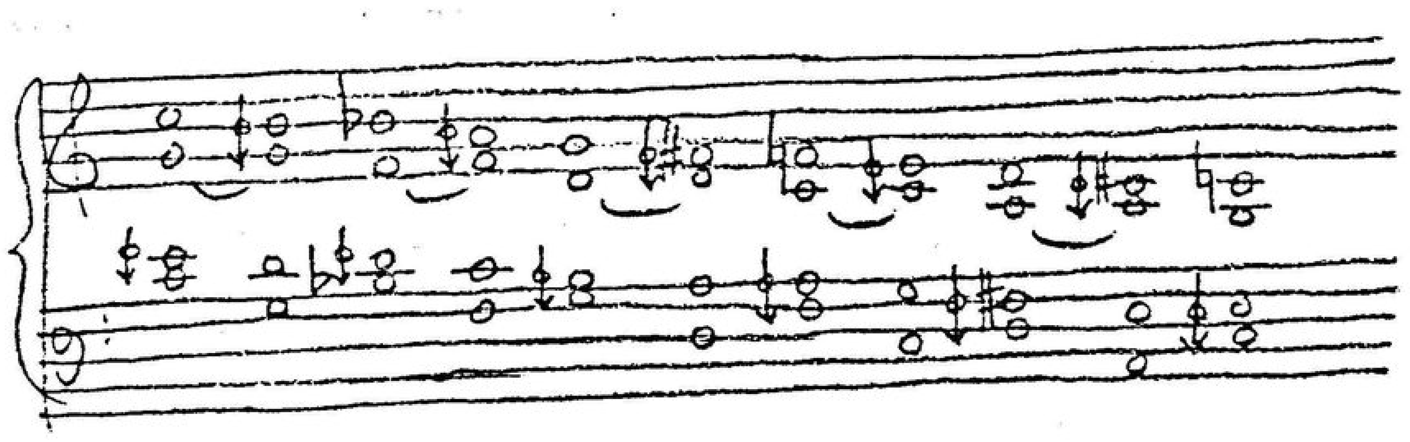
Very interesting things happen when the five triads of p. 14 are played through the circle of fifths — i.e, so that their roots are C, F, Bb, D, G, C (remembering that Bb D is also a fifth in the downward direction). We have



, and if this is continued so that the roots repeat in the same manner, the top voice is the 10-note symmetric mode itself. For comparison, a similar progression within the 8-note symmetric mode (12-note equal) is as follows:

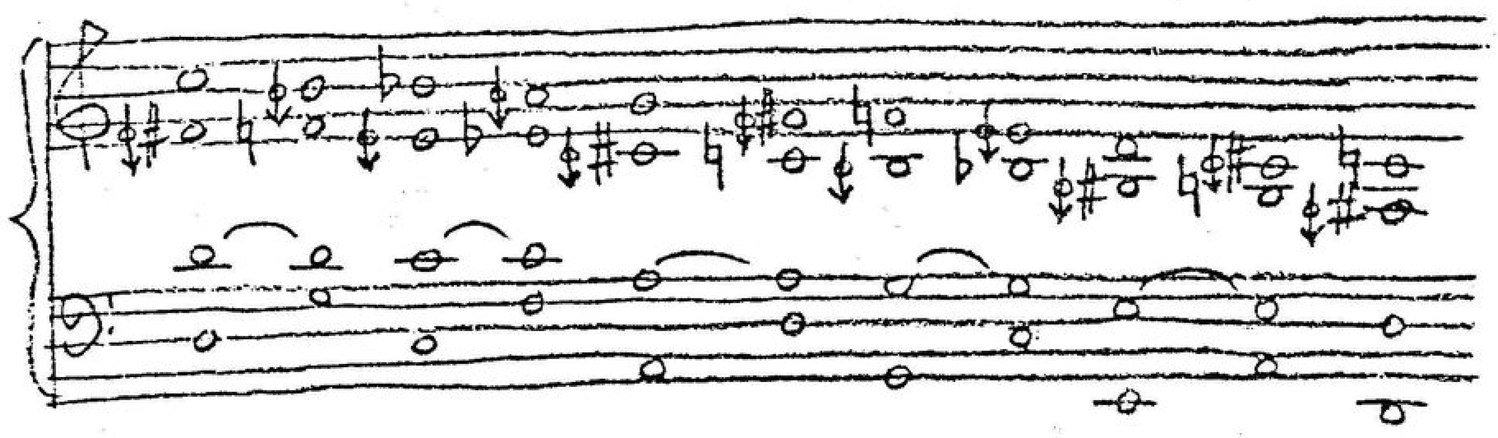


Another sequential harmonization of the 10-note symmetric mode, using the five major triads of p. 14, is the following:



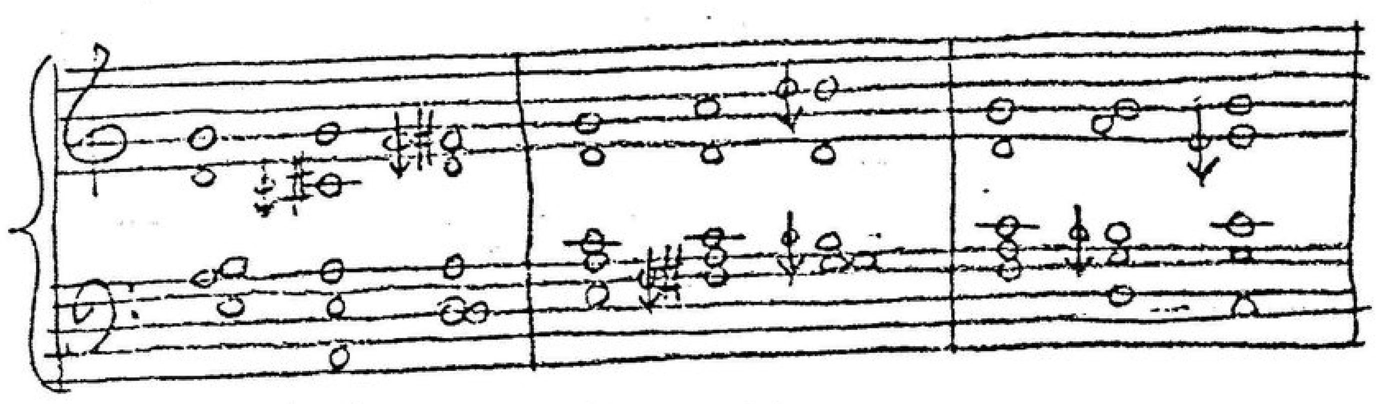
This one has a very pleasantly surprising effect, as does the following, made out of five minor triads:

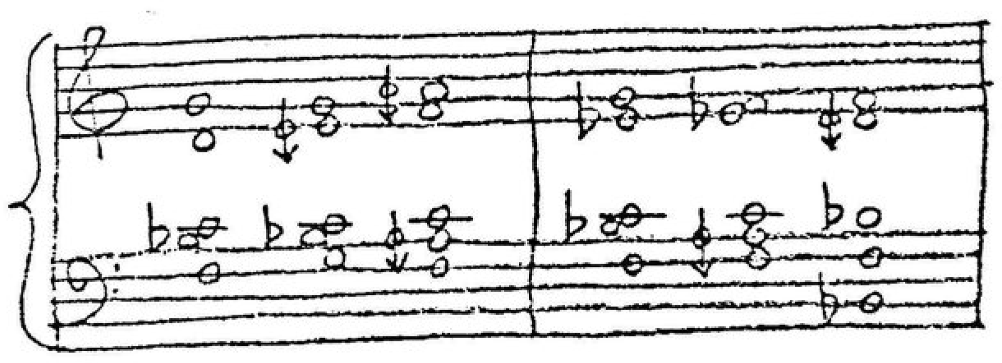
In addition to G B♮↓ D F, the 10-note symmetric mode possesses four other out-of-tune, but not objectionable dominant seventh chords: viz. C E♮↓ G Bb, F A♮↓ C Eb, A C♯↓ E G, and D F♯↓ A C. The 10-note symmetric mode may be harmonized sequentially by an endless succession of these chords, with their roots always falling by a fifth of 720 cents:



The resemblance between the above and a well-known similar progression in 12-note equal tuning shows how adaptable the notation can be to new situations.

Of especial interest is the harmonic behavior of the 5-note equal division itself. To my ear, this sounds like a very weak, but intensely discordant version of a subdominant harmony. But all similarity ends here, since the equal division ensures that there exist five harmonically identical resolutions into five different keys, construing each of the five notes in turn as the subdominant root. (This is just like the chromatic behavior of a diminished seventh chord in 12-note equal tuning, wherein each of the four notes may be construed as a leading tone, by virtue of the division into four equal parts.) Like the 12-note equal diminished seventh, which takes on four different spellings in four different keys, the 5-note equal division takes on the five different spellings of p. 12 in five different keys:





The relation of triads and their notation to the 6-note symmetric mode is no less interesting. A triad will have the same tuning — major third $= \frac{5}{15}a$, minor third $= \frac{4}{15}a$ — in the 6-note symmetric mode if we make

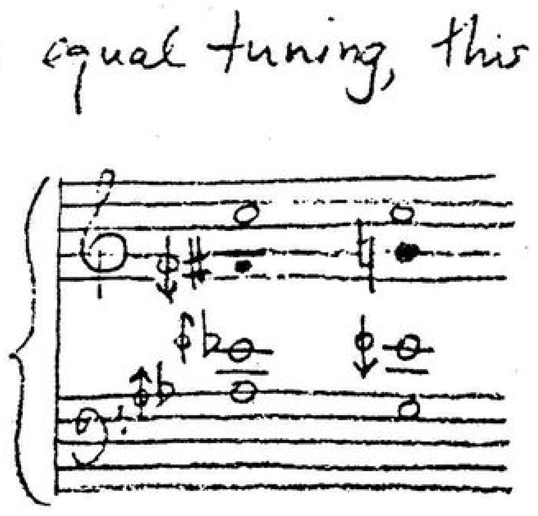
make the larger interval equal to $\frac{4}{15}a = 320$ cents (a closer approximation of a pure minor third than the 300 cents of 12-equal), and the smaller equal to $\frac{1}{15}a = 80$ c
The mode includes positions 0, 4, 5, 9, 10, 14, 15. The notatio
according to the scheme of p. 13, is C, D♮↑, E♮↓, G, G♮↑, B♭↓,
If we remember that G♮↑ is equivalent to A♭↑, then the "Prokofiev progression" of p. 4 becomes

Note that the equivalence A♭↑ – G♮↑ in the above version is the equivalence Ab – G# in 12-note equal tuning.

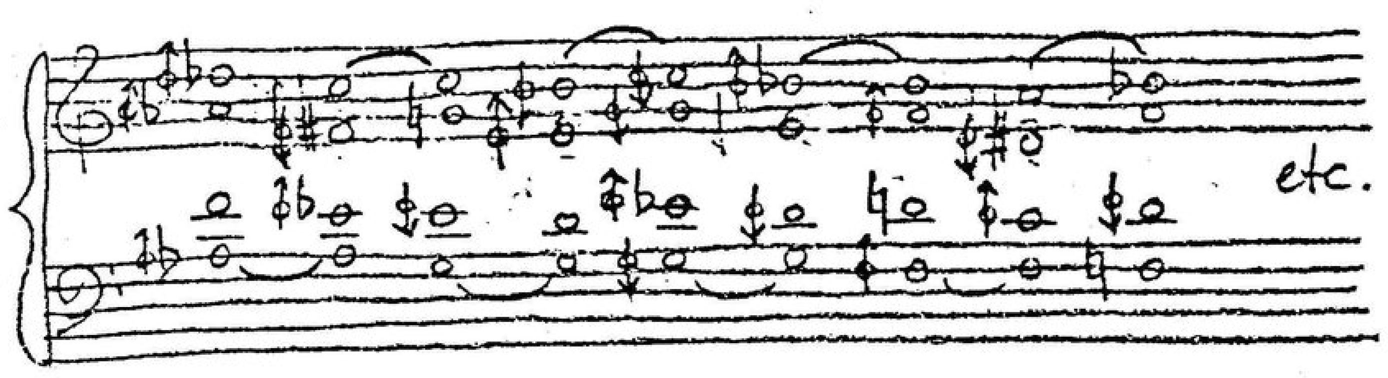
In 12-note equal tuning, the progression from Ab major to C major may be regarded as a German 6th, missing the F#, to I, —

with each part, save the soprano, moving by $\frac{1}{12}a$. And, a
is well known, the first harmony may also be the dominan
seventh in Db, by virtue of the equivalence of F# and Gb

In 15-note equal tuning, this becomes



with the alto moving up $\frac{1}{15}a$, the tenor up $\frac{1}{15}a$, and the bass down $\frac{1}{15}a$. This tuning of the first chord is the same as the dominant seventh in Db‡ major, and makes possible a modulating sequence where the dominant seventh in the old key becomes the German sixth in the next key. The first few repetitions of the pattern appear as follows:



Tonal compositions within 15-note equal tuning, they would make extensive use of embellished versions of [illegible] are chromatic or non-tonal sequences have so far been rather less interesting in their effect. I would expect extensive uses of the 6 and 10-note symmetric modes, along with modulating sequences that return to the initial harmony after three, five, or fifteen repetitions.

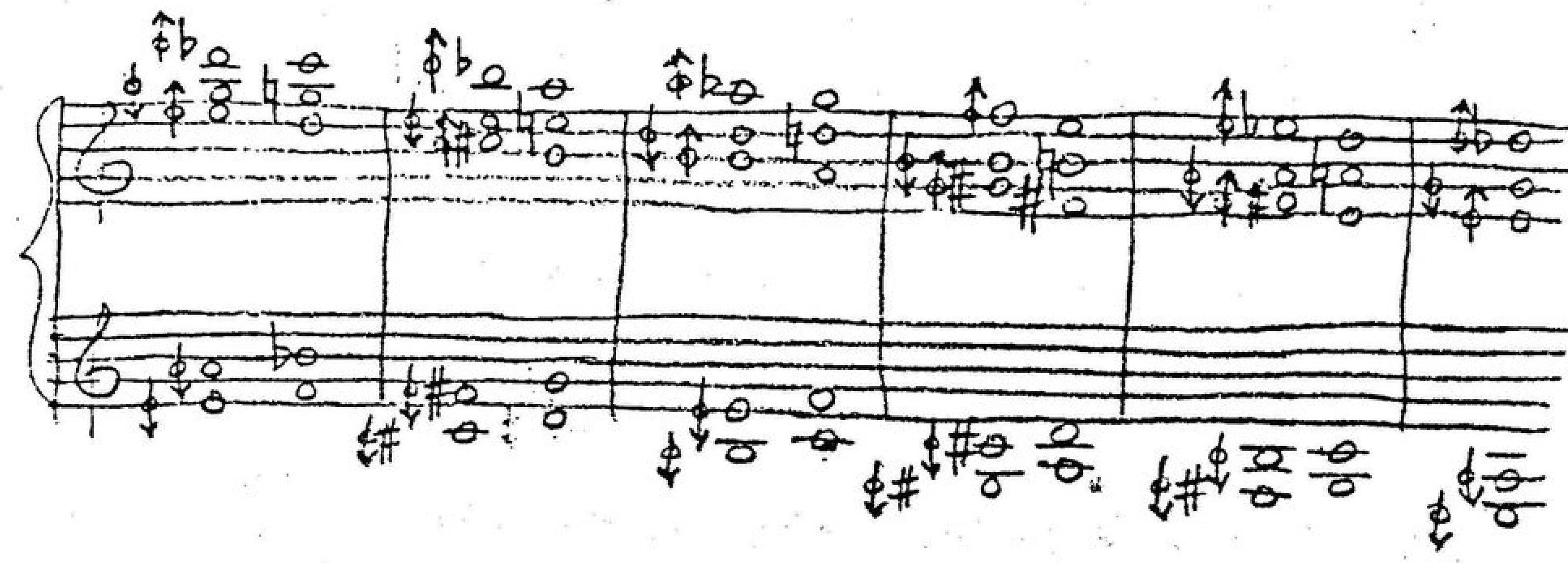
Generally speaking, there may exist several classes of modulating sequences within a given equal tuning, and these are distinguished by how many keys they traverse before returning to the initial key. For example, in 12-note equal tuning, modulating sequences may traverse two, three, four, six, or all twelve keys, but no other number of keys. As an example, the "Rimsky-Korsakov sequence" of p. 4 traverses three keys. It is not necessary for a sequence traversing three keys to be confined to the six-note symmetric mode, however. For examples, see Schubert's Piano Sonata in A major, D. 959, first movement, bars 28-34; also his String Quartet in G major, Op. 161, first movement, bars 170-176. As an example of a modulating sequence traversing four keys, see Tchaikovsky's Sixth Symphony, first movement, bars 31-37, and also 204-210. Sequences that traverse six keys are seldom continued through all possible repetitions, and the same is true for those that traverse 12 keys. Sequences that do not define keys may also return to the initial harmony after 2, 3, 4, 6 or 12 repetitions. I think it best to call such sequences <u>exact</u> <u>transposition</u> sequences. The "Liszt sequence" of p. 4 is a case involving four repetitions. See also Schoenberg Klavierstück Op. 11, Nr. 1, bar 30; although there are only two occurrences of the sequential pattern, the original harmony (AG#) would recur at the sixth repetition.

It is no coincidence that the numbers 2, 3, 4, 6, and 1 are all factors of 12, or that 3, 5, and 15 are factors of 15. Generally, in an equal tuning of X notes, if an exact transposition sequence first returns to the initial harmony after y repetitions, then y must be a factor of X. In consequence, if the number of notes X is prime, all exact transposition or modulating sequences, whatever their nature, must repeat X times, or traverse all keys, before returning to the initial harmony. This is a disappointing state of affairs with regard to 19-note equal tuning, which contains recognizable diatonic scales that are fairly well in tune.

In 15-note equal tuning (as in any equal tuning), it is not necessary that sequences traversing three or five keys should use only notes within the symmetric modes. For example, if the sequence of p. 16 is changed so that the first chord is minor and the second major, with this alternation continuing, the result is a modulating sequence using all 15 notes, that traverses only five keys.

Exact transposition sequences do, in general, impart a palpable musical order to just about any progression whatever. One interesting example embellishes the 5-equal harmony with chromatic upper and lower neighbors

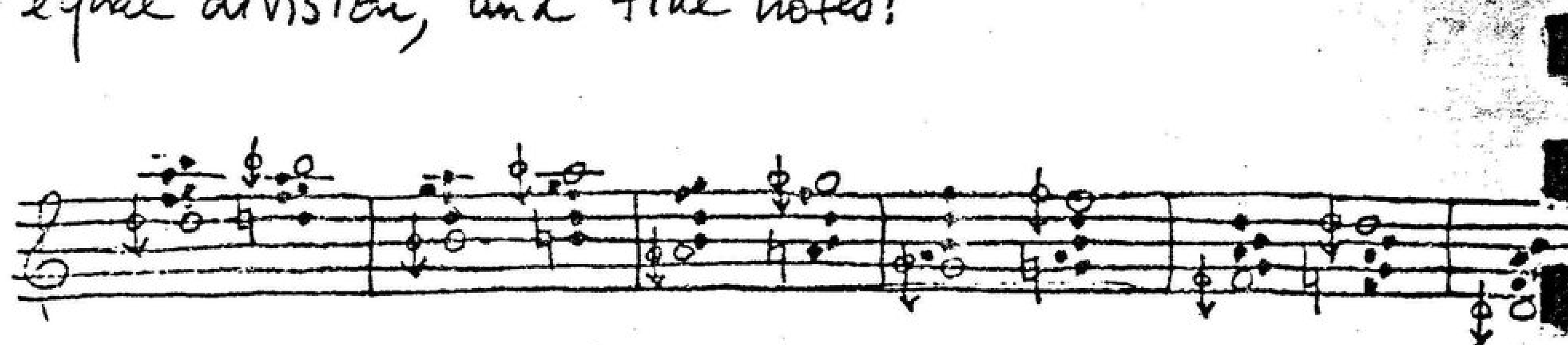
in four of the five parts:



Another interesting case arises from extending concepts which lead to familiar sequences within the 8 and 6-note symmetric modes. Consider in 12-note equal tuning the following:

The first chord in each measure consists of three of the 4-equal division, with the lower note $\frac{1}{12}$a below the fourth 4-equal note. The second chord also consists of three of the 4-equal, with the top note $\frac{1}{12}$a below. (The 4-equal notes are solid.) Furthermore, the second chord in each measure is an inversion of the chord that follows. This progression, all within the 8-note symmetric mode, has an analog in the 6-note symmetric mode, as follows:

It can be seen at once that the arrangement is much like the preceeding, with one note less, and involving the 3-equal divion. Clearly an analog exists within the 10-note symmetric mode, involving the 5-equal division, and five notes:



What makes the 6 and 8-note versions interesting is that they make a different connection between harmonies that arise from other considerations — triads in the case of 6 notes, and dominant sevenths in the case of 8 notes. If we rewrite the 10-note version, changing only the notation, but not the sound, as follows, —

it can be seen that the chords are major dominant ninths in five different keys — D, C, Bb, G, and F.

More in a couple of weeks.

It has been observed before that 19-note equal tuning contains notatable recognizable diatonic scales that are also acceptable. It is interesting to show why by a means that has other broad applications. Generally a recogniz[able] diatonic scale consists of five major seconds and two mino[r] seconds in a particular order. Now call a major second w (for "whole step"), and a minor second h (for "half step"). If the scale starts and ends with the tonic, the order of major and minor seconds within that octave is w, w, h, w, w, [w, h.] Calling an octave a, we have at once $5w+2h=a$. No further stipulation regarding the relative sizes of w and h is necessary, save that both intervals must be ascending, with the major seconds greater than the minor seconds. In oth[er] words, we have $0<h<w$; and if all intervals are measure[d] in cents, it follows that $5w+2h=1200$. The arrangement w, w, h, w, w, w, h, where $5w+2h=1200$ and $0<h<w$ depicts an entire family of recognizable diatonic scales. Specification of any one of a number of other elements will describe a specific member of the family; what is wanted is another element that is readily comprehensible to musical intuition. An especially useful element is the